Luento 1.

Puhtaan siirtymän hankkeiden rahoitus

Markku Kaustia

Hannes Gebhard Professor of Finance and Insurance, Department of Finance, Aalto University

We approach the course themes through the lens of corporate finance

Corporate finance is a major branch of finance theory

Lays the foundation for the analysis of corporate investment projects

Developed in the US in 1950 – 1980

– "Imported" to Europe in 80's, to Finland in 90s

Aiheet

Rahoitusvaihtoehtojen vertailumenetelmät

Pääoman kustannus

Oma- ja vieras pääoma, velkarakenne

Pääomasijoittaminen, venture capital, listautumisanti

Kansainvälinen rahoitus ja riskeiltä suojautuminen

Miten EU-taksonomia vaikuttaa rahoitukseen (Katja Tuokko, TEM)

+ Harjoitustyön rooli

Components affecting the value of an investment project

Cost of investment (–)

Cash flow from investment (+)

Cost of capital (–)

Each of these have several key driving factors

Corporate Finance in one slide

$$PV = \sum_{t=1}^{T} \frac{E(X_t)}{(1+r_t)^t}$$

Think of X as the cash flow generated by the firm

You can affect X and r

- What real investments to make and how
- Capital structure of the firm

You can affect the E, the expectation operator

- For example, what outsiders (like financial markets) expect about the cash flows
- E.g., signaling by dividend policy

The present value formula

$$PV(X,r) = \frac{E(X)}{1+r}$$

'*E*' is the expected value operator, frequently dropped in notation, I drop it as well

The present value of a future uncertain cash flow X is obtained by discounting its expected value

The discount rate r reflects

- The timing of X, so r applies from current time (0) to when X is received
- The opportunity cost of capital, given the risk of X (but for now, just think of it as some interest rate)

Two disciplines by the present value formula

$$PV(X,r) = \frac{X}{1+r}$$

X: "Financial accounting"

$1+r$: "Financial economics"

Green projects by the present value formula

$$PV(X,r) = \frac{X}{1+r}$$

X: **Profits and costs**
Regulatory mandated investments, subsidies

$1+r$: **Cost of capital**
Subsidies, tax breaks, green finance instruments, green investment clienteles

Rahoitusvaihtoehtojen vertailumenetelmät

Aalto University

Time value of money

Depositing for a single period

Choice between \$100 and \$110 right now (kind of easy…)

Choice between \$100 today and \$110 in 1 year?

Calculating Future Value (*FV*)

– Bank account with 6% interest per year
– After one year, you have \$100 + interest
– Interest = 0.06 * \$100 = \$6, so you have \$106

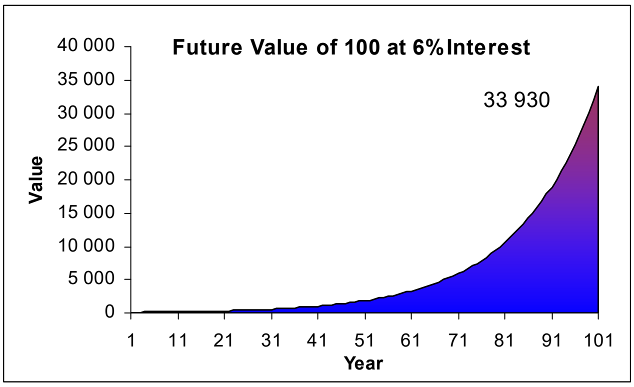
Investing for more than one period

- How much do you have after 2 years?
  106 + 0.06 * 106 = 112.36
  - You earn 36 cents as "interest on interest"
  - This is compound interest
- How much do you have after 3 years? *Always* assuming compound interest…
  112.36 + 0.06 * 112.36 = 119.10

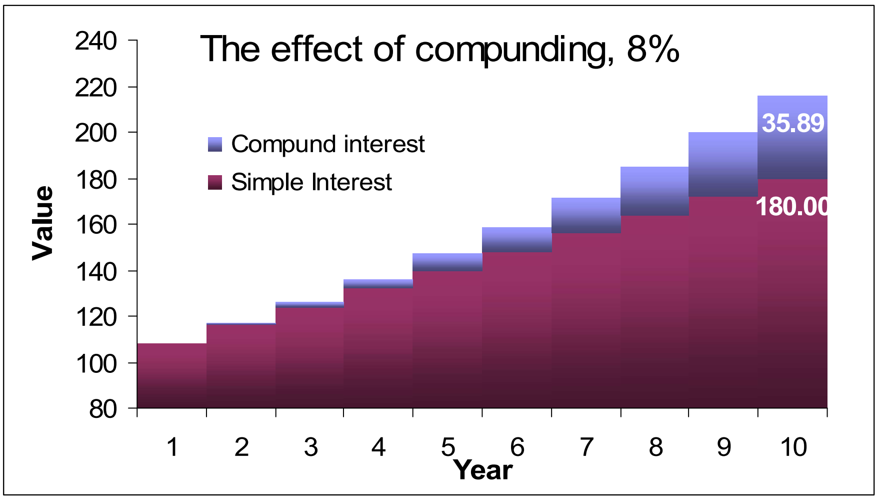
Future Values with compound interest

Time	Amount
0 or Today	100.00 = PV
1 year	106.00 = PV (1 + .06)
2 years	112.36 = PV (1 + .06) (1 + .06)
3 years	119.10 = PV (1 + .06) (1 + .06) (1 + .06)
4 years	126.25 = PV $(1 + .06)^4$
5 years	133.82 = PV $(1 + .06)^5$
6 years	141.85 = PV $(1 + .06)^6$
...	...
100 years	33,930.20 = PV $(1 + .06)^{100}$

Year	Value
0	100
1	106
2	112
3	119
4	126
5	134
10	179
20	321
50	1 842
100	33 930



Deposit 100 for 10 years at 8%



Generally, after t years you have:

$$FV_t = X(1+r)^t$$

where *FV* = Future value, *X* = sum of money invested now, *r* = interest rate, *t* = number of years

In one of the more controversial land deals in history, ...


General Minuit

Dutch colony general Peter Minuit purchased the island of Manhattan in 1626 from the Lenape tribe in exchange for trade goods valued at 60 guilders ($24)

400 years later, had the Indians deposited the money at 5%, they would have:

$$\$24(1+0.05)^{400} = \$7.18 \times 10^{9}$$

Is 5% realistic? Let's see the result with 0.5% rate:

$$\$24(1+0.005)^{400} = \$176.45$$

Present value and discounting

Present value

- What sum of money today is worth a given sum in the future?

Discounting

- The act of converting future sums of money to present value

The single-period case

Choice between $100 today and $110 in 1 year (interest rate 6%)

- Calculate the Future Value of $100: The value of 100 in 1 year: 106
- Or, calculate the Present Value (PV) of the $110

What sum of money today (X) is worth $110 in one year?

$$X \times 1.06 = 110$$

$$\Rightarrow X = \frac{110}{1.06} = 103.77$$

Present value (PV) of $1 received in one year

$$PV \text{ of } \$1 = \frac{\$1}{(1+r)}$$

r = discount rate

The present value of a sum of money Y received next year is the sum of money X which would grow to be Y in one year

$$PV \text{ of } Y = \frac{Y}{(1+r)} = X$$

Earlier we talked about an interest rate, but the previous slide calls r discount rate?

The *discount rate* provides a "rate of exchange" between today's money and future money

It can take many forms, depending on the problem at hand

- Various kinds of interest rates, expected stock return, interest rate plus a margin…
- It is different from the purchasing power of money, which declines by the rate of infaltion

Why different discount rates? Consider these investment returns

Venture capital investments, 15 - 20%

"We require a return of 10% for our investments" (CEO of a large pulp and paper company)

"7-8% is considered a good long-term return for real estate investments"

Swiss bank account? 0.25%

What is an adequate return from one investment, may not be adequate for another one

An appropriate discount rate reflects the opportunity cost of capital

In general, what is an opportunity cost? (vs. out-of-pocket cost)

The opportunity cost of capital is the return that the capital could earn elsewhere with similar risk

Present value for multiple periods

Suppose tuition fees for a private four-year college will be $300,000 in 20 years from now

How much do expecting parents need to set aside now to be able to send their child there? Assume investing return of 7.5% pa.

We could try out different starting values and see what takes us to $300 thousand

Or, just calculate the present value:

$$PV = \frac{300,000}{1.075^{20}} = 70,623.94$$

For a given discount rate – the longer the time to wait for the money, the lower the present value

$$\frac{100}{(1.06)^4} = 79.21 \qquad \frac{100}{(1.06)^5} = 74.73$$

For a given time to wait – the higher the discount rate, the lower the present value

$$\frac{100}{(1.06)^4} = 79.21 \qquad \frac{100}{(1.07)^4} = 76.29$$

Present versus future value

Relation between present value (PV) and future value (FV)

$$FV_t = PV(1+r)^t$$

$$PV = FV_t \frac{1}{(1+r)^t}$$

FV = future value
PV = present value
r = discount rate
t = years

We know either PV or FV at the outset, depending on the problem

The present value formula can be expressed in many ways

$$PV \text{ of } X = \frac{X}{(1+r)^t} = X\frac{1}{(1+r)^t} = X(1+r)^{-t}$$

They are all mathematically equivalent

In the second formulation, the latter part, i.e., $1 / (1 + r)^t$ is called the discount factor

- Exchange rate between future and present money

Determining the discount rate

"Determining the rate of return" is more accurate terminology in this context

Suppose I invest $17 in the stock of Bank of America

I sell it in three years for $25

To make things simple for now, assume the stock pays no dividends

What is my rate of return for the 3 year period?

$$\frac{\$25 - \$17}{\$17} = \frac{\$25}{\$17} - 1 = 47.06\%$$

What is my rate of return per year? (annualized return)

Let's see if we can use the present value formula

$$PV = \frac{FV}{(1+r)^t}$$

– Do we know present value or future value?

– Actually, we know both: PV = \$17, FV = \$24

– What about the timeframe? Three years

– So, the only unknown variable is r

Plug in the numbers $\$17 = \frac{\$25}{(1+r)^3}$

$$17(1+r)^3 = 25$$ **Both sides multiplied by $(1 + r)^3$**

$$(1+r)^3 = \frac{25}{17}$$ **Both sides divided by 17**

$$1+r = \sqrt[3]{\frac{25}{17}}$$ **A cube root has been taken of both sides**

$$r = 1.1372 - 1 = 0.1372$$ **Subtracted 1 from both sides, which gives the answer 13.72% pa.**

This tells us that there must be some value of r that makes this equality hold

Recall the algebraic correspondence between powers and roots

$$\sqrt{x} = x^{\frac{1}{2}}, \quad \sqrt[3]{x} = x^{\frac{1}{3}}, \quad \sqrt[y]{x} = x^{\frac{1}{y}}$$

The present value formula relates future values to present values

It has four components: PV, FV, r, and t

Whenever three of them are known, the fourth can be figured out

In problems that call for determining present value, we must know (or estimate) FV, r, and t

In problems that call for determining the rate of return we must already know PV, and we also need to know FV and t

Finding the number of periods

This is the hardest variant

If we invest in a risky start-up earning 25% return per year, when can we expect to double our money?

Let's make a guess. Three years?

$$PV(1+r)^t = 1 \times 1.25^3 = 1.953$$

In three years, each dollar we invested will have turned into 1.95 dollars, but not quite into two

Try solving this with the present value formula

$$PV = \frac{FV}{(1+r)^t}, \quad 1 = \frac{2}{1.25^t}$$

$$\Rightarrow 1.25^t = 2$$

This equation will give us the correct answer if we only could solve it for *t*. We can, taking logarithms from both sides:

$$\log(1.25^t) = \log 2$$

$$t \log(1.25) = \log 2$$

$$t = \frac{\log 2}{\log(1.25)} = \frac{0.30103}{0.09691} = 3.1063$$

Discounted cash flow analysis

Future value with multiple cash flows

1. What are the amounts and when are they paid, or received?

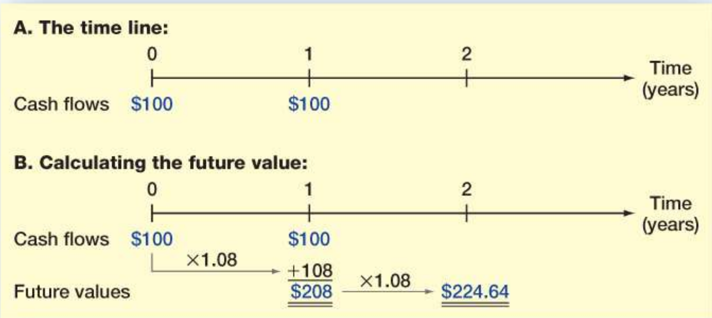
2. Do the math

At first #2 may seem like the hard part

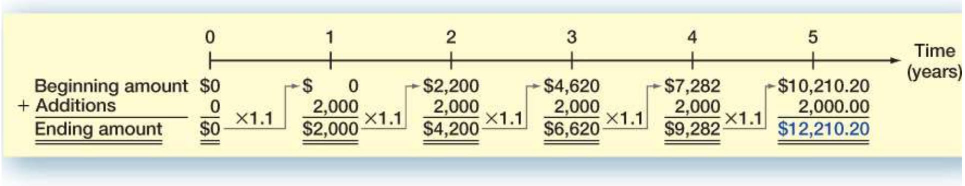
In business practice #1 is the hard part

Math is usually straightforward and always at least doable exactly, but

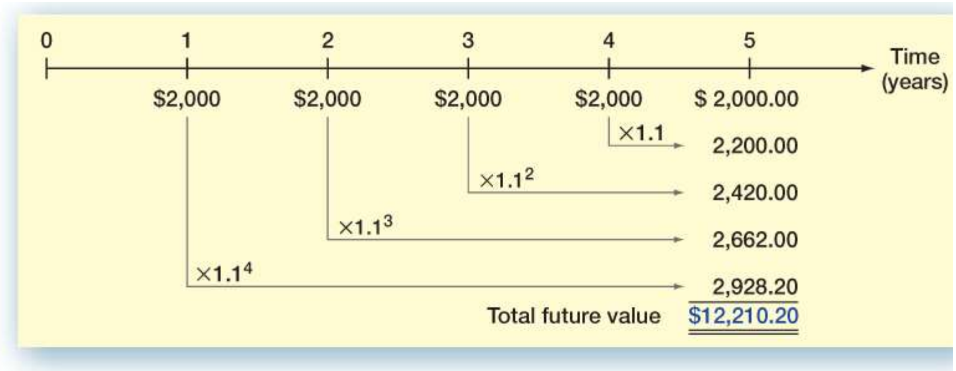
- Estimating cash flow is ambiguous, uncertain
- Needs assumptions and judgment



- You deposit $100 today in an account paying 8%
- In one year, you will deposit another $100
- How much will you have in two years?



- The future value of $2,000 invested at the end of each of the next five years, at 10%
- Compound the investment one period at a time, rolling that amount to the next year

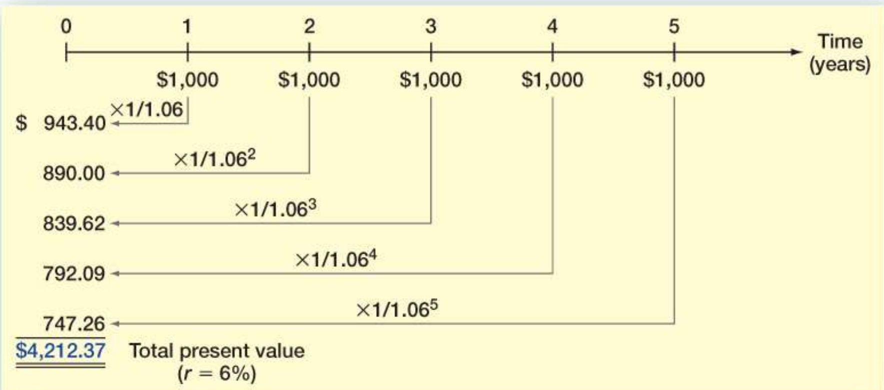


- Alternative technique
  - Calculate the future value of each individual cash flow
  - Sum up the results

Present value with multiple cash flows

Calculating present values is the mirror image of calculating future values

- Instead of "moving" the cash flows forward, we "move" them backward in time
- Instead of multiplying, we divide by $(1 + r)^t$



(or $4,212.36)

- Starting in a year from now, you need to make 5 annual payments of $1,000
- What amount of capital do you need to set aside to cover those payments if you earn 6% interest?

Let’s check that this is correct

Calculate the forward value each year

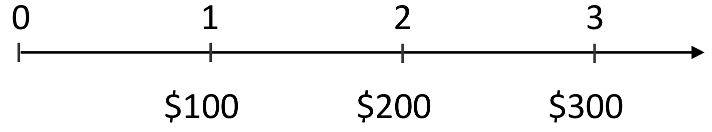
Deduct the payment, and repeat…

	Capital	
t	at t	at t + 1, before payment
0	4,212.36	4,465.10
1	3,465.10	3,673.01
2	2,673.01	2,833.39
3	1,833.39	1,943.39
4	943.39	999.99
5	-0.01	

A note about cash flow timing

The default assumption is that cash flows occur at the end of each period

For example, “A three-year investment has a first-year cash flow of $100, a second-year cash flow of $200, and a third-year cash flow of $300”



In most cases the periods are years, but can also be months or quarters

To handle fractions of years, modify the exponent *t*

E.g., PV of $100 received in six months, 7%

$$\frac{100}{(1.07)^{0.5}} = 96.67$$

E.g., PV of $100 received in June 1 next year (assume today is Sep 1), 7%

$$\frac{100}{(1.07)^{0.751}} = 95.05$$

– How many days is it? 274 days

– What fraction of a year is that? 274/365 = 0.751

Discounted cash flow analysis

- Special (but typical) cases of repeating cash flow patterns

Valuing level cash flows: annuities and perpetuities

Special formulas for periodically repeating cash flows

Annuity: A level cash flow that goes on for a fixed number of periods and then stops

Perpetuity: A level cash flow that goes on forever

Present value for annuity cash flows

Present value of cash flow *C* for *t* years

$$= C \times \left[\frac{1 - \text{Present value factor}}{r} \right]$$

$$= C \times \left[\frac{1 - \left[\frac{1}{(1+r)^t} \right]}{r} \right]$$

= C x A, where A is the Annuity Present Value Factor

PV of $500 for 10 years, at 7%

$$\$500 \times \left[\frac{1 - \left[\frac{1}{(1.07)^{10}} \right]}{0.07} \right] = \$3{,}511.79$$

Intermediate result for checking

$$\frac{1}{(1.07)^{10}} = 0.508349$$

Finding the payment

You get an auto loan

- Loan amount $15,000
- Term 48 months
- 4.8% annual interest

What is your monthly payment?

The key identity: Present value of the monthly payments must equal the loan amount

For monthly payments we need a monthly interest rate: 4.8% / 12 = 0.4%

$$PV = C \times A, \quad \Rightarrow C = \frac{PV}{A}$$

$$A = \left[\frac{1 - \left[\frac{1}{(1+r)^t} \right]}{r} \right] = \left[\frac{1 - \left[\frac{1}{(1.004)^{48}} \right]}{0.004} \right] = 43.59425$$

$$C = \frac{\$15{,}000}{43.59425} = \$344.08$$

Let's check intuition

Your total monthly payments are 48 x $344.08 = $16,515.94

How much more is this compared to the loan amount?

- $16,515.94 / $15,000 – 1 = 10.1% more

Interest rate was 4.8%, so why do you end up paying "only" 10.1% more than the borrowed amount? While for four years, 4 x 4.8% ≈ 20%?

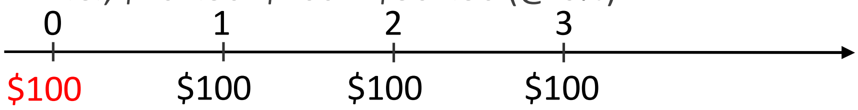
- Because didn't borrow the full amount for four years, you started paying back right away

A note about annuities due

With an ordinary annuity the cash flows occur at the end of each period (e.g., a car loan)

An annuity due has cash flows at the beginning of each period (e.g., a lease)

- Same as having an ordinary annuity + an extra cash flow in the beginning
- E.g., an annuity due of $100 with 4 payments has the same PV as an ordinary 3-year annuity + $100, i.e., $267.30+$100 = $367.30 (@ 6%)



You can convert any series of cash flows to one (imaginary) lump sum

You can then treat that as any other single cash flow

This is very useful