

Luento 2.

Puhtaan siirtymän hankkeiden rahoitus

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Aiheet

Investointilaskemat

Pääoman kustannus

Oma- ja vieras pääoma, velkarakenne

Pääomasijoittaminen, venture capital, listautumisanti

Kansainvälinen rahoitus ja riskeiltä suojautuminen

Miten EU-taksonomia vaikuttaa rahoitukseen (Katja Tuokko, TEM)

+ Harjoitustyön rooli

Discounted cash flow analysis

- Common special cases of repeating cash flow patterns

Perpetuity

A level cash flow that continues forever

- It's nominal value is infinite, no matter how small the cash flow
- Its present value is finite, no matter how large the cash flow

Present value of a perpetuity = $\frac{C}{r}$

where C is the periodic cash flow, first paid at the end of the first period

How do we have such a simple formula for an infinite sequence of present values?

$$\frac{C}{r} = X$$

$$X = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots$$

$$X(1+r) = C + \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots \quad \left| \begin{array}{l} \text{multiplied} \\ \text{by } (1+r) \end{array} \right.$$

$$X(1+r) = C + \underbrace{\frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots}_{= \text{PV of perpetuity} = X}$$

$$X(1+r) = C + X$$

$$X + rX = C + X$$

$$rX = C$$

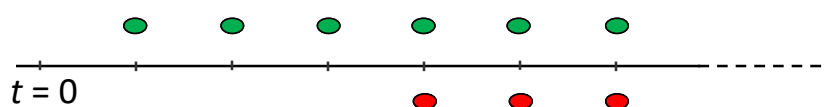
$$X = \frac{C}{r}$$

PV for annuity cash flow formula is derived as a difference between two perpetuities

Perpetuity A has its first payment at the end of the first period

Perpetuity B has its first payment one period after the annuity ends

E.g., consider an annuity of three cash flows



It is equivalent to $A - B$

Example: Retirement package choice from the US military 1992 downsizing program

Voluntary separation package was offered to the personnel

O-3 level officer with 9 years of service gets a choice of

- \$46,219 lump sum immediately
- An annuity of \$7,703 for 18 years, paid in the beginning of each year

Captain Johnson wants to retire from the Army and is pondering which offer to take...

Present value of an annuity due covering 18 years

= present value of an annuity for 17 years
+ a payment right now

Let's use a discount rate of 10%

$$PV = 7,703 \times \left[\frac{1 - \left[\frac{1}{(1.1)^{17}} \right]}{0.1} \right] + 7,703 = 69,493.03$$

Which option should Captain Johnson take?

What if Capt. J needs the money right now?

Assuming she can get an 18-year loan with an interest rate of 10%

- With annuity option, she immediately receives the first \$7,703
- Borrow \$40,000 as an annuity bank loan, now have \$47,703 in cash to use
- Annual payment of her new loan at end of each year is \$4,877; This is more than covered by the beginning of year incoming annuity of \$7,703

She should still select the annuity package

In general, future liabilities can be financed when...

$$PV(\text{Liabilities}) \leq \text{Current assets} + PV(\text{Future income})$$

What level of income will just support the financing of liabilities?

$$PV(\text{Liabilities}) = \text{Current assets} + PV(\text{Future income})$$

...and solve for unknown income

Note: this assumes that current assets and future income can be invested to produce a return equal to the discount rate

Growing perpetuity

A cash flow that starts at level C , then grows by $g\%$ every period, and goes on forever

$$PV = \frac{C}{r - g}$$

Growing annuity

A cash flow that starts at level C , then grows by $g\%$ every period, and goes on for a total of t payments

Its present value is derived from the difference of two growing perpetuities

This starts at time 1,
grows by g , and goes
on forever

This starts at time $t + 1$,
grows by g , and goes
on forever

$$\frac{C}{r-g} - \frac{(1+g)^t \times C}{r-g} \times \frac{1}{(1+r)^t}$$

Growing annuity formula versions

$$\frac{C}{r-g} - \frac{(1+g)^t \times C}{r-g} \times \frac{1}{(1+r)^t} = C \left[\frac{1}{r-g} - \frac{(1+g)^t}{(r-g)(1+r)^t} \right]$$

$$= C \left[\frac{1}{r-g} - \frac{(1+g)^t / (1+r)^t}{r-g} \right]$$

$$= C \left[\frac{1 - \frac{(1+g)^t}{(1+r)^t}}{r-g} \right] = C \left[\frac{1 - \left(\frac{1+g}{1+r} \right)^t}{r-g} \right]$$

Example: Foundation leasing a house for 12-month periods

Example: A foundation is leasing a house for 12-month periods at a time

Assume all the year's rent is payable in the beginning of the year

At first the lease is for \$20,000 per year; Then each year for new tenants and the rent is increased by 3%

PV of the house (as an investment asset), assuming discount rate of 7%?

It's an infinite and growing income stream, i.e., a growing perpetuity

The formulas assume that the first cash flow occurs at the end of the first period

This is not the case here, so we just add the first year's payment separately

$$PV = \$20,000 + \frac{\$20,000 \times 1.03}{0.07 - 0.03} = \$535,000$$

Financing instruments

- A quick first look on debt capital instruments

Loan types and loan amortization

Three basic types of loans

- Pure discount loans
- Interest-only loans
- Amortized loans

Pure discount loans

The simplest form

The borrower receives money today, and repays a single lump sum at some time in the future

Examples: T-bills, interbank certificates of deposits (CD's)

Interest-only loans

The borrower pays interest each period, but no principal

The entire principal (the original loan amount) is paid at some point in the future when the loan matures

Examples: government bonds, most corporate bonds

Amortized loans

The borrower pays parts of the loan principal over time

The most common way is to have the borrower make a fixed payment of principal every period

The per period interest payment is calculated for the principal outstanding at the start of the period

Amortization table, auto loan example	Month	Payment, \$		Balance, \$
		Interest	Principal	
	1	60.00	284.08	14,715.92
	2	58.86	285.22	14,430.70
	3	57.72	286.36	14,144.34
interest rate	4	56.58	287.50	13,856.84
0.4% per month;	5	55.43	288.65	13,568.18
payment	6	54.27	289.81	13,278.37
\$344.08 per	7	53.11	290.97	12,987.40
month	8	51.95	292.13	12,695.27
	9	50.78	293.30	12,401.97
	10	49.61	294.47	12,107.50
beginning	11	48.43	295.65	11,811.84
balance \$15,000	12	47.25	296.83	11,515.01
	⋮			
	⋮			

NPV rule of investment decisions

CEO and now vs. later

Current versus future benefit

A firm has \$370K in cash and nothing else (to make it simple for now)

The CEO comes across a great opportunity to invest it in a new factory

It will produce a sure (risk-free) \$420K next year

Should the CEO invest in this project?

- Pros: Increases future wealth
- Cons: Requires \$370K today, so can't pay it as dividends to shareholders

CEO and now vs. later

The CEO discovers there are two types of shareholders in her company, each own 50%

Patient type

- Favors projects that pay off in the long-term
- E.g., trusts, retirement savings plans, who don't need cash until later

Impatient type

- Wants the company to quickly pay out all the cash in dividends
- E.g., a recent retiree, who needs cash today for consumption

CEO and now vs. later

How does the CEO decide whether to invest in the factory?

1. Invest in the factory to satisfy patient types
2. Do not invest in the factory and pay out the cash as dividend to satisfy impatient types

Assume the borrowing and lending rate is 5%

CEO and now vs. later

Consider the impatient owner type

- If the firm does not invest in the factory, he will have \$185K today
- If the firm invests, he can borrow \$200K against the future payoff of the project (of which he owns 50%, or \$210K),
- He is \$15K wealthier compared to not investing

CEO and now vs. later

Consider the patient type

- If the firm invests, she will have \$210K in a year
- If the firm does not invest, she receives the dividend, lends it out at 5%, and will have $1.05 \times 185K = 194,250$ in a year

Makes sense that the patient type is better off, they didn't need current consumption in the first place

But the impatient type is also better off

CEO and now vs. later

Both types are better off when the firm invests

Q: So the firm should always invest as much as possible?

A: No. In this case it was a value-adding investment, and you should always invest in such cases. But investing in value-reducing investments is not good. If that's all you have, then you should just pay the dividend instead.

Q: So what are these value adding investments?

A: Those with $NPV > 0$

Net Present Value (NPV)

Corporate real investments are usually not marketable securities

Assume “as if” they were: if the cash flow stream was securitized and traded on the market, its market price would be equal to its PV

The value proposition of corporations

Based on trying to acquire / set up these real investments with an initial cost $<$ PV(cash flow)

The difference is called Net Present Value (NPV)

$$\text{NPV} = -\text{acquisition cost} + \text{PV}(\text{cash flow})$$

The positive amount of NPV is the value creation from the ability to do the investment

[Financial securities on the other hand are thought to have NPV = 0]

NPV rule for Investment

Accept a project (only) if its NPV is positive

- Creates wealth that can be spent now or in future

CF profile and NPV equation of a prototypical project

$$NPV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \frac{C_4}{(1+r)^4} + \dots$$

C_0 is the initial setup cost, cash flow (C_i) can be different each year, and the project goes on for many years

Estimating net present value

1. Estimate expected future cash flows (“Cash is King”)
2. Estimate the required return for the project, given its risk
3. Compute $NPV = PV(\text{Benefits}) - PV(\text{Costs})$

Nice things about the NPV rule

Whereas one could increase profits by tricks like cutting dividends and investing the cash in T-Bills, you cannot create shareholder wealth that way (NPV = 0)

NPV focuses on cash flows, immune to disagreements about accounting rules

- Whereas a decision that improves profits in one accountant's books can reduce them in another's

NPV rule reflects opportunity costs and risk of the project

A closely related useful concept: Internal Rate of Return (IRR)

First leave discount rate unspecified, then set:

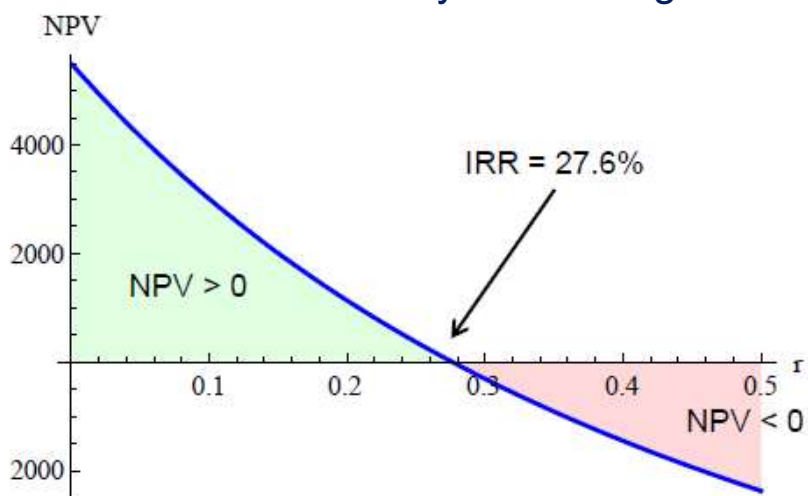
$$0 = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \frac{C_4}{(1+r)^4} + \dots$$

IRR is the discount rate that achieves NPV = 0

- Interpretation as the annualized return on investment

Often, but not always, a value creating project is one that has IRR > cost of capital

NPV is often monotonically decreasing in r



Graph, such as this, showing the NPV of a project for different discount rates is called the NPV profile

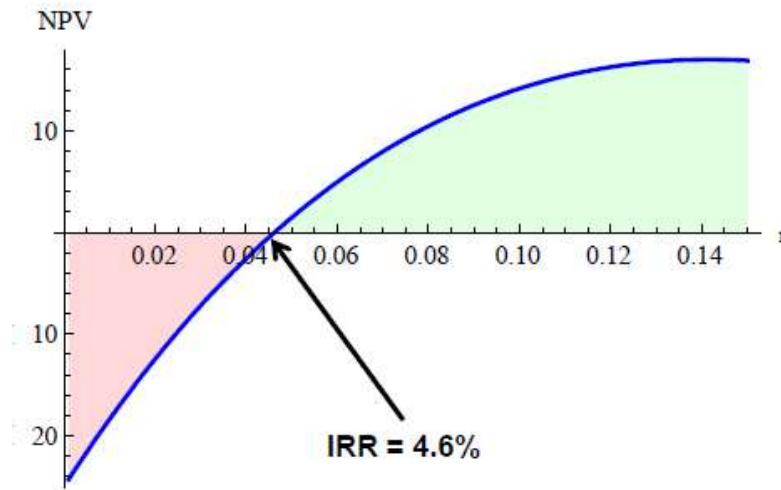
IRR is very useful for calculating returns, but can be misleading for selecting projects

E.g., when some future cash flows are negative, IRR can give the wrong answer

Example: A new strip mine costs \$925 today; Future CF will be \$1,000, \$1,400 in years 1 to 2, and in year 3 you have to clean up the site and pay CF = \$1,500 (a cost)

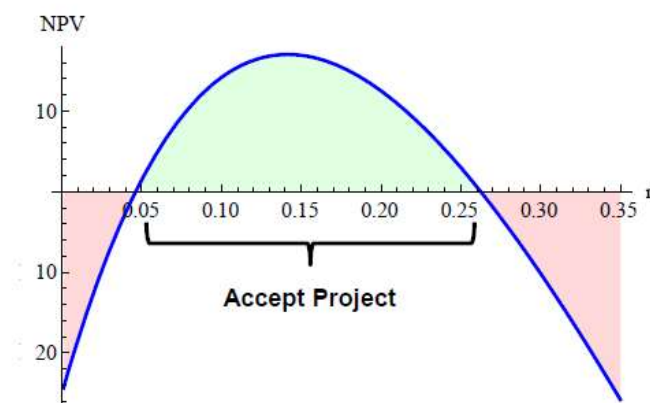
Here IRR = 4.6%, so is this a good project if cost of capital < 4.6%?

Hmm, is this project's value *increasing* in r ?



Zoom out to a bigger picture...

Here NPV is non-monotonic in r , there's another IRR at 26.3%



Often projects have an initial set-up cost C_0 , and positive cash flow thereafter, so IRR equation is:

$$0 = -C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \dots + \frac{C_T}{(1+r)^T}$$

Often high r is like receiving a high rate of return on the project for your given C_0 which alternatively could earn the discount rate r if you had invested it elsewhere

But in the strip-mining example, one of the future cash flows is negative

It's as if we need to "pay back" a return r on our own investment F

- High r could be bad for the project if the repayment is large (\$1,500 in this case)

So, when r increases, NPV could go up or down

More generally: when the signs (+/-) of the cash flows switch more than once, there can be multiple IRR's

[It's also possible that there is *no IRR at all*, try: $CF_1 = -210$, $CF_2 = 455$, $CF_3 = -270$]

Example of 3 (mutually exclusive) projects

A plot of land is on sale for 100. There are three possible uses

Farming: Steady cash flow (25) from now until year 60, starting this year

Forestry: Trees take 10 years to mature (CF = 0), Then a cash flow (60) for 50 years

Mining: High cash flow (50) for the next 5 years, then the land will be useless

Project cash flow table

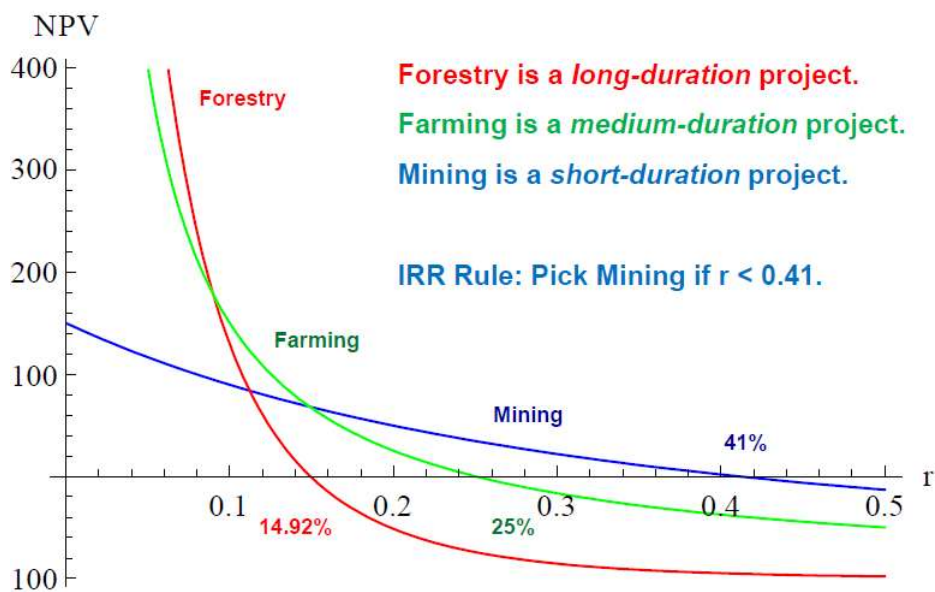
Year	Farming	Forestry	Mining
0	-100	-100	-100
1	25	0	50
2	25	0	50
3	25	0	50
4	25	0	50
5	25	0	50
6	25	0	0
...	25	0	0
10	25	0	0
11	25	60	0
12-60	25	60	0

Modeling

Farming: 60-year annuity; $NPV = -100 + 25/r * (1 - 1/(1+r)^{60})$; IRR = 25%

Forestry: 50-year annuity, but delayed by 10 years;
 $NPV = -100 + 60/r * (1 - 1/(1+r)^{50}) * 1/(1+r)^{10}$; IRR = 14.9%

Mining: 5-year annuity; $NPV = -100 + 50/r * (1 - 1/(1+r)^5)$; IRR = 41%



Normally: just go with NPV in decision-making; **In subsidized finance:** $NPV < 0$ project can add value, may need to use IRR rule

General themes with cash flow durations

At low discount rates, long-duration projects such as forestry do best; Tech stocks are like forestry: you must wait to harvest

At high discount rates, short-duration projects like mining do best; Tobacco stocks are like mining: High current cash flow, but it may dwindle/stop staticsoon

At intermediate discount rates, intermediate-duration projects like farming do best; Utility stocks are like farming: Steady cash flow

Which project to choose in the example?

Choice is not to accept vs. reject each project one by one; It was picking one of 4 options: 3 projects, or nothing

IRR rule is a ranking on returns: Not always a sensible method of ranking

NPV rule is a ranking on wealth creation; The best project generates most wealth today

– But to use it, you need to know your cost of capital

Pääoman kustannus

Aalto University

Cost of capital as an opportunity cost

Your vacation example: (1) out-of-pocket cost, (2) opportunity cost, (3) public externality cost

Opportunity cost of capital is the return that the capital would earn elsewhere

- **Classic def.:** with similar risk
- **Sustainable finance def.:** with an acceptable combination of return, risk, and externalities

An appropriate discount rate reflects the opportunity cost of capital

Key elements of cost of capital

How much debt, how much equity? (i.e., capital structure)

Any tax advantage of debt?

Required returns on equity and debt

Project-specific risk considerations

Concepts to cover: Firm value, cash flow calculations, leverage effect