

Helsinki

Handbook for Bilingual Education

Mathematics in grades 1–9 in
Large-Scale Bilingual Education

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Language outcomes in mathematics

Indexed by the Finnish National Core Curriculum

Introduction

The need for a handbook on subject-specific language goals arises from the national curriculum, which requires that students' development in subject-specific academic language be assessed according to the goals set. The first version of this handbook was created in 2019 as a collaborative work between the cities of Espoo, Helsinki and Vantaa.

In the spring of 2023, this work was taken back under construction and a team of subject teachers as well as classroom teachers started to work on the handbook, focusing on clarifying the curricular content goals, setting linguistic goals and providing scaffolding ideas and concrete examples of how to help students reach the set goals. Soon we decided to make several handbooks on different subjects.

This handbook for mathematics in bilingual education was ready for a round of comments in February 2024. Maths teachers from Helsinki, Pori, Turku, Oulu and Kotka commented on the penultimate version of the handbook, which was ready for layout in April 2024. And here it is!

We are very grateful for the financial support we received from the Finnish National Board of Education for the development projects Helsinki oppii kielillä 5 and 6, which funded the production of this handbook.

I would like to thank all the teachers who commented on the handbook along the way!

Above all, I would like to thank the members of the project group for their hard work, commitment and enthusiasm in producing this handbook. It has been a great pleasure to work with you, Anna, Janne and Jukka. East or west, you are the best ;)

Helsinki, August 2024

Jenni Kohl



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Checklist for sound classroom practices

I ensure that my classroom environment

is safe and friendly.

☆☆☆☆☆

is rich and supports learning.

☆☆☆☆☆

incorporates visual scaffolding methods.

☆☆☆☆☆

I foster learner autonomy

by giving learners choices to make.

☆☆☆☆☆

by explicitly teaching learning skills and strategies.

☆☆☆☆☆

by negotiating on decisions about the learning process.

☆☆☆☆☆

During my lessons, my learners often

learn cooperatively.

☆☆☆☆☆

discuss mathematical content in English.

☆☆☆☆☆

discuss mathematical content in Finnish.

☆☆☆☆☆

read written mathematical content in English.

☆☆☆☆☆

read written mathematical content in Finnish.

☆☆☆☆☆

write mathematical content in English.

☆☆☆☆☆

write mathematical content in Finnish.

☆☆☆☆☆

I collaborate with various colleagues in planning my lessons.

☆☆☆☆☆

The intended outcomes for learning

content are explicit and visible to my learners.

☆☆☆☆☆

language are explicit and visible to my learners.

☆☆☆☆☆

I often challenge my learners by

using extensive academic language orally.

☆☆☆☆☆

using extensive academic language in written form.

☆☆☆☆☆

making them use academic Finnish.

☆☆☆☆☆

making them use academic English.

☆☆☆☆☆

I use various methods to scaffold

content outcomes for my students.

☆☆☆☆☆

language outcomes for my students.

☆☆☆☆☆

During my lessons, the learners have been exposed to

vocabulary regarding C1 in Finnish.



vocabulary regarding C1 in English.



vocabulary regarding C2 in Finnish.



vocabulary regarding C2 in English.



vocabulary regarding C3 in Finnish.



vocabulary regarding C3 in English.



vocabulary regarding C4 in Finnish.



vocabulary regarding C4 in English.



vocabulary regarding C5 in Finnish.



vocabulary regarding C5 in English.



vocabulary regarding C6 in Finnish.



vocabulary regarding C6 in English.



For mathematics books, we have used the following:

- | | | |
|-------|----------------------------------|----------------------------------|
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |

For additional instructional material, we have used the following:

- | | | |
|-------|----------------------------------|----------------------------------|
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |
| _____ | <input type="checkbox"/> Finnish | <input type="checkbox"/> English |

1st–2nd grade

C1 Thinking skills

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C1 for grades 3–6, and C1 for grades 7–9.

Content outcomes

C1 Thinking skills

The pupils

- are provided with opportunities to find similarities, differences, and regularities.
- compare, classify, and place objects in order and identify causal relationships.
- practise examining mathematical situations from different viewpoints.
- begin familiarising themselves with the basics of programming by formulating and testing step-by-step instructions.

Language outcomes

Listening skills

The pupils practise:

- listening to others.
- identifying core mathematical terminology and vocabulary from spoken Finnish and English.
- understanding simple oral assignments and instructions in spoken Finnish or English.
- understanding and following oral instructions in spoken Finnish and English.
- identifying the key information in orally given word-based tasks in Finnish and in English.

Reading skills

The pupils practise:

- identifying key mathematical terminology and vocabulary in written Finnish and English.
- identifying the key information in written word-based tasks in Finnish and in English.

Speaking skills

The pupils practise:

- using appropriate terminology to describe similarities, differences, and regularities (rf. Key words in this section).
- explaining their thought process when comparing, classifying, and placing objects in order, and when identifying causal relationships (rf. Key words in this section).
- rephrasing assignments in their own words.
- giving brief, clear instructions in Finnish and English.
- discussing, sharing, and exchanging ideas, strategies, and ways of thinking.

Writing skills

The pupils practise:

- writing down their answers.
- writing and spelling mathematical terminology and vocabulary in Finnish and English.

Attachments and links

Add within 20 visually, by Khan Academy,
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Key words

Classifying (Luokittelu)

to group (ryhmitellä), to sort (järjestää), to compare (vertailla); to be the same as (olla sama kuin), similar to (samanlainen/samankaltainen/yhtenevä kuin), different from (eri/erilainen kuin), to belong in (kuulua johonkin); greater than (suurempi kuin), less than (pienempi kuin), to be equal to (olla yhtä suuri kuin), to equal (olla yhtä suuri kuin), equals (on yhtä suuri kuin).

Programming (Ohjelmointi)

a program (ohjelma), to program (ohjelmoida), a step (askel), forward (eteenpäin), backwards (taaksepäin), (a set of) instructions (ohjeet)

Solving problems (Ongelmanratkaisu)

to prove (todistaa), a proof (todistus), to explain (selittää), an explanation (selitys), a pattern (kuvio/toistuva ominaisuus), a sequence (jono), colo(u)r (väri), size (koko)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping adopted concepts, terminology, vocabulary, and notations visible in the classroom, while introducing new visual aids next to previous related ones.

Using AI to transform data into convenient forms:

- E.g. to form a **row-by-row handout** based on the terminology lists in this document: “Could you reformat the following list so that all the terms are on their own separate rows?”
- E.g. to form a row-by-row handout based on the terminology lists in this document in **either language only**: “Could you reformat the following list so that all the terms are on their own separate rows and that only the words in Finnish are included?”

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Offering students with sentence diagrams they can fill in to classify objects.

Providing visual aids to help forming strings of commands and instructions when programming.

C2 Numbers and operations

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C2 for grades 3–6, and C2 for grades 7–9.

Content outcomes

C2 Numbers and operations

Operations are performed using natural numbers. Addition and subtraction operations are made concrete when applying them in different contexts.

It is ensured that the pupils

- are familiar with the connection between numbers, numerals, and digits.
- expand their understanding of numbers by counting, perceiving, and estimating quantities.
- practise their number sequencing skills and the ability to compare and place numbers in order.
- examine the properties of numbers, such as parity, multiples, and division by two.

The pupils

- become acquainted with the decompositions of numbers from 1 to 10.
- are guided in using numbers appropriately in different situations, expressing quantity, order, and measurement results, and performing operations.
- familiarise themselves with the principles of the decimal system through concrete models.
- develop their addition and subtractions skills first in the number range 0–20 and then in the range of 0–100.
- practise different mental arithmetic strategies to improve their arithmetic skills.
- learn to use the commutative and associative properties when carrying out additions.
- are guided to understand the concept of multiplication through concrete examples, and they learn the multiplication tables of 1–5 and 10.
- apply the commutative property in multiplication and familiarise themselves with the associative property of multiplication.

The instruction creates a basis for understanding division and the connection between multiplication and division. The concept of fraction is introduced by learning to divide an entity into equal parts.

Language outcomes

Listening skills

The pupils practise:

- identifying core mathematical terminology and vocabulary from spoken Finnish and English.
- understanding simple oral assignments and instructions in spoken Finnish or English.
- understanding and following oral instructions in spoken Finnish and English.
- identifying the key information in orally given word-based tasks in Finnish and in English.

Reading skills

The pupils practise:

- identifying key mathematical terminology and vocabulary in written Finnish and English.
- identifying the key information in written word-based tasks in Finnish and in English.
- translating numerals into numbers and vice versa
 - e.g. 🍎🍎🍎 being equal to 3
- reading and writing numbers in natural language and vice versa
 - e.g. 9 being nine and 13 being thirteen

Speaking skills

The pupils practise:

- translating numerals into numbers and vice versa
 - e.g. 🍎🍎🍎 being equal to 3
- reading and writing numbers in natural language and vice versa
 - e.g. 9 being nine and 13 being thirteen
- converting operations on natural numbers to and from natural language using correct terminology
 - e.g. $2 \cdot 3$ being multiplication: being the product of 2 and 3, being 2 and 3 multiplied, and being 2 times 3
- using the verbs *to count*, *to calculate*, and *to find* to fit the context
 - e.g. to count the apples, to calculate the sum in euros, to find the missing number
- explaining their thought process when operating with natural numbers.
 - e.g. “7 plus 6 equals 13 because 7 plus 3 equals 10 and 10 plus 3 equals 13.”
- rephrasing assignments in their own words.

Writing skills

The pupils practise:

- writing numbers and their names in Finnish and English.
- writing symbols for operations.
- illustrating their answers to back them up with shapes, figures, and numbers.

Attachments and links

Add within 20 visually, by Khan Academy,
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Key words

Operations (Laskutoimitukset)

to count (laskea / luetella), to calculate (laskea laskutoimitus), to find (selvittää), to add (laskea yhteen), addition (yhteenlasku), a sum (summa), to subtract (vähentää), subtraction (vähennyslasku), difference (erotus), to take away (ottaa pois), plus (plus), minus (miinus), to multiply (kertoa), multiplication (kertolasku), times (kertaa), multiplied by (kerrottuna jollakin), a multiplier (kertoja), a product (tulo)

double (kaksinkertainen), twice as many/much as (kaksi kertaa niin monta/paljon kuin), triple (kolminkertainen), three times as many/much as (kolme kertaa niin monta/paljon), to increase by (kasvaa jonkin verran), to decrease by (vähetä jonkin verran)

Numbers and digits (Luvut ja numerot)

a number (luku), numbers 0–20 (luvut 0–20), ones (ykköset), tens (kymmenet), a digit (numero), two-digit (kaksinumeroinen), numbers 0–100 (luvut 0–100), hundreds (sadat)

Ordinal numbers (Järjestysluvut)

first (ensimmäinen), second (toinen), third (kolmas), fourth (neljäs), ...

Classifying and comparing numbers (Lukujen luokittelu ja vertailu)

the number line (lukusuora), a dash (viiva lukusuoralla), more (enemmän), less (vähemmän), fewer (vähemmän), greater than (suurempi kuin), (the) most (eniten), (the) least (vähiten), left over (ylijäävä/yli jäänyt), equal to (yhtä suuri kuin), to equal (olla yhtä suuri kuin)

a multiplication table (kertotaulu), number bonds (hajotusparit), ten bonds (kymppiparit), to double (kaksinkertaistaa), doubles (tuplat), to halve (puolitittaa), halves (puolikkaat), a fact family (käänteiset laskutoimitusperheet), even (parillinen), odd (pariton), a group of (joukko jotakin)

in total (yhteensä), altogether (yhteensä), in all (yhteensä) lots of (paljon jotakin), a lot of (paljon jotakin)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping numerals and their names visible in the classroom throughout.

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Offering students with sentence diagrams they can fill in to read the operations using natural language.

C3 Geometry

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C4 for grades 3–6, and C5 for grades 7–9.

Content outcomes

C3 Geometry

The pupils

- improve their skills in perceiving the three dimensional environment and observing elements of plane geometry in it.
- practice using the concepts of direction and location.
- are guided to examine, identify and also build and draw objects and plane figures.
- are guided to find and name properties that are used to classify objects and plane figures.
- practise measuring, and they are guided to grasp the principle of measurement.
- discuss the quantities length, mass, volume, and time, and practise using their respective units of measurement: the key units are metres and centimetres, kilogrammes and grammes, and litres and decilitres.
- practise reading time on a clock face, and practise using units of time.

Language outcomes

Listening skills

The pupils practise:

- identifying terminology and vocabulary related to geometry and measuring from spoken Finnish and English.
 - e.g. “You will need your ruler in the next assignment.”
 - e.g. “Draw a square on the paper in front of you, please.”
 - e.g. “We will start the assignment in 15 minutes, what time is it then?”
 - e.g. “It is half past two.” versus “Kello on puoli kolme.”

Reading skills

The pupils practise:

- identifying key mathematical terminology and vocabulary in written Finnish and English.
 - e.g. “Use your ruler to measure the sides of the square below.”
 - e.g. “What time is it?” or “What is the time?” in contrast to “Mitä kello on?”
 - e.g. “Write down the time shown on the clock face above.”
- identifying shapes and figures by their written names in Finnish and in English.
 - e.g. “The one on the left is a circle, and I think this is a pentagon.”

Speaking, writing, and illustrating skills

The pupils practise:

- explaining key properties of geometrical shapes and figures using their own words
 - e.g. “A square has all sides equal.”
 - e.g. “There are three sides in this triangle.”
- explaining their thought process when carrying out measurements.
 - e.g. “I drew this with a ruler (and) I measured 2 cm. I forgot to write it down (though).”

- explaining their answers, and supporting them with relevant shapes and figures
 - e.g. “This one is a square (because) It is the same as this one.”
- rephrasing assignments in their own words.
 - e.g. “I need a ruler to draw a rectangle. I will draw four lines.”

Attachments and links

Telling time on a number line, by Khan Academy,
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Telling time on a clock, by Khan Academy,
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Key words

Geometry (Geometria)

on (top of) (päällä), under (alla), above (yläpuolella), below (alapuolella), to the left of (vasemmalla jostakin), to the right of (oikealla jostakin), in front of (jonkin edessä), behind (jonkin takana)

a shape (kuvio/muoto), a figure (kuvio/muoto/ kuvituskuva), a square (neliö), a quadrilateral (nelikulmio), a circle (ympyrä), a triangle (kolmio), two-dimensional / 2D (kaksiulotteinen)

a rectangle (suorakulmio), a polygon (monikulmio), a pentagon (viisikulmio)

an angle (kulma), a vertex (many vertices) (kärki), a side (kylki), a line (suora), a line segment (jana), a grid (ruudukko)

a solid (avaruuskappale), three-dimensional / 3D (kolmiulotteinen), a cube (kuutio), a sphere (pallo), a pyramid (pyramidi), a cylinder (lieriö), a cone (kartio)

a mirror image (peilikuva), to be similar to (olla yhdenmuotoinen jonkin kanssa), to resemble (muistuttaa jotakin)

Measuring (Mittaaminen)

to measure (mitata), a measurement (mittaus), to carry out / conduct a measurement (mitata), a result (tulos), a value (arvo), to read the result / value / answer (lukea tulos / arvo/ vastaus), to write down / to log (kirjata / kirjoittaa ylös)

a ruler (viivain / viivotin), a measuring tape (mittanauha), a metre / meter m (metri m), a centimetre / centimeter cm (senttimetri cm), long (pitkä), short (lyhyt), tall (pitkä / korkea), high (korkea), wide (leveä), deep (syvä), the height (korkeus), the width (leveys), the depth (syvyys), a kilometre / kilometer km (kilometri km), a gram(me) g (gramma g), a kilogram(me) kg (kilogramma kg), a litre / liter l (litra l), a decilitre / deciliter dl (desilitra dl), to weigh (punnita), a weight (paino), a length (pituus), a (weighing) scale (vaaka)

a clock (kello), a clock face (kellotaulu), to tell the time (katsoa kelloa), an hour (tunti), the hour hand (tuntiviisari),

a minute (minuutti), the minute hand (minuuttiviisari), o'clock (tasatunti / tasan), a second (sekunti), a second hand (sekuntiviisari), a quarter (of an hour) (vartti / neljäsosatunti), clockwise (myötäpäivään), anti-clockwise / counter-clockwise (vastapäivään)

a price (hintaa), a cost (kustannus), money (raha), an euro (euro), a cent (sentsi)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping numerals and their names visible in the classroom throughout.

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Offering students with sentence diagrams they can fill in to read the operations using natural language.

C4 Data processing and statistics

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C5 for grades 3–6, and C6 for grades 7–9.

Content outcomes

C4 Data processing and statistics

The pupils

- begin to develop their ability to collect and store information on interesting topics.
- draw and interpret simple tables and bar graphs.

Language outcomes

Listening skills

The pupils practise:

- identifying core mathematical terminology and vocabulary from spoken Finnish and English.
- understanding simple oral assignments and instructions in spoken Finnish or English.
- understanding and following oral instructions in both Finnish and English.
- identifying the key information in orally given word-based tasks in Finnish and in English.

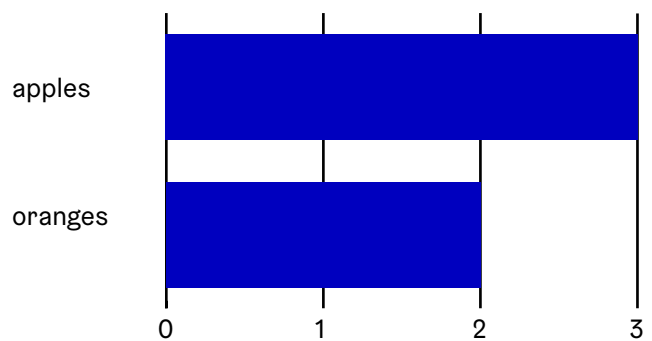
Reading skills

The pupils practise:

- identifying key mathematical terminology and vocabulary in written Finnish and English.
- identifying the key information in written word-based tasks in Finnish and in English.
- counting the number of elements in a group, and storing data in a table and representing it in graph format
- e.g. 🍏🍏🍏🍊🍊 being equal to:

Simple table that tells amounts of apples and oranges.

fruit	number
apples	3
oranges	2



Speaking skills

The pupils practise:

- reading data in table and bar graph format.
 - e.g. “There are three apples and two oranges.”
- comparing data groups in table and bar graph format.
 - e.g. “There is one apple more than there are oranges.”

Attachments and links

Unit: *Data*, by Khan Academy,
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Key words

Structuring data (Tietojenkäsittely)

data (tiedot / data), a data set (tietue / datajoukko),
a data group (tietue / datajoukko), a bar graph
(pylväsdiagrammi), a pictogram (havainnollistuskuva),
a table (taulukko), a row (rivi), a column (sarake), a cell
(solu), a chart (kaavio/taulukko)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology,
vocabulary, and notations through repetition.

Output scaffolding

Breaking down assignments into easy-to-follow,
step-by-step instructions.

Offering students with sentence diagrams they
can fill in to interpret graphs and read data in
table format.

3rd–6th grade

C1 Thinking skills

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C1 for grades 1–2, and C1 for grades 7–9.

Content outcomes

C1 Thinking skills

The pupils

- develop their skills in finding similarities, differences, and regularities.
- improve their skills in comparing, classifying, and ranging objects, systematically searching for alternatives, and observing causal relationships and connections in mathematics.
- plan and execute programs in graphic programming environments.

Language outcomes

Listening and reading skills

The pupils practise:

- understanding and following oral instructions in both Finnish and English.
- identifying the key information in orally given word-based tasks in Finnish and in English.

The pupils practise:

- solving oral and written problems, tasks, and assignments by:
 - identifying the context of the task based on its notations and mathematical terminology.
 - specifying the goal of the task.
 - predicting a range for possible solutions for the task.
 - reflecting if a solution fits the task setting.

The pupils practise:

- translating blocks of code into natural language commands, and vice versa.

Speaking and writing skills

The pupils practise:

- expressing themselves orally and in written form by, for example:
 - asking questions relevant to the topic, task or assignment.
 - explaining their thought process behind their solution to a problem concisely and using relevant and correct terminology and vocabulary.
- backing up their reasoning or solution with concise, relevant arguments, clearly written intermediate steps, and appropriate, understandable figures.

The pupils practise:

- spelling and writing mathematical terminology and vocabulary in both Finnish and in English.

The pupils practise:

- describing similarities, differences, and regularities with appropriate terminology (rf. Key words in this section).
- explaining their thought process when comparing, classifying, and placing objects in order, and when identifying causal relationships (rf. Key words in this section).
- describing the structure of code in a graphic programming environment.
- explaining the purpose of blocks of code in a graphic programming environment.
- giving constructive feedback on a pupil's, a teacher's or an AI's thought process, and comparing it to their own approach.
- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- assessing, evaluating, and reasoning whether the obtained value fits the assignment, task or problem.

Attachments and links

Speaking frame to support comparing numbers.

	<p>is equal to equals on yhtä suuri kuin</p> <p>is greater than on suurempi kuin</p> <p>is less than on pienempi kuin</p> <p>is different from on eri kuin</p> <p>is the same as on sama kuin</p> <p>is similar to on samankaltainen kuin</p> <p>belongs in the same group with kuuluu samaan ryhmään kuin</p>	
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Cognitive discourse functions

Kognitiiviset diskurssifunktiot



Key words

Classifying and comparing numbers (Lukujen luokittelu ja vertailu)

to group (ryhmitellä), to sort (järjestää), to compare (vertailla, verrata); to be the same as (olla sama kuin), similar to (samankaltainen kuin), different from (eri kuin / erilainen kuin), to belong in (kuulua johonkin); greater than (suurempi kuin), less than (pienempi kuin), equal to (yhtä suuri kuin), to equal (olla yhtä suuri kuin)

Programming (Ohjelmointi)

a program (ohjelma), to program (ohjelmoida), a block of code (pätkä koodia), to code (koodata), a step (askel), forward (eteenpäin), backwards (taaksepäin), (a set of) instructions (ohjeet), to estimate (arvioida), to debug (debugata / etsiä virhe), a bug (bugi / virhe), an algorithm (algoritmi), a sprite (hahmon kuvake), a command (käsky), to command (käskeä / käskyttää), an input (syöte), an output (tuloste); if–then–else (jos – niin – muutoin), until (kunnes), repetition (toisto), repetitive (toistava), a loop (silmukka); a variable (muuttuja), a selection (valinta), to select (valita)

Solving problems (Ongelmanratkaisu)

to prove (todistaa / osoittaa), a proof (todistus / osoitus), to explain (selittää), an explanation (selitys), a pattern (kuvio / toistuva ominaisuus), a sequence (jono), colo(u)r (väri), size (koko), to be plausible (olla mahdollinen)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping adopted concepts, terminology, vocabulary, and notations visible in the classroom, while also introducing new visual aids alongside related ones.

Using AI to transform data into convenient forms:

- E.g. to form a **row-by-row handout** based on the terminology lists in this document: “Could you reformat the following list so that all the terms are on their own separate rows?”
- E.g. to form a row-by-row handout based on the terminology lists in this document **in either language only**: “Could you reformat the following list so that all the terms are on their own separate rows and that only the words in Finnish are included?”

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Offering pupils with sentence diagrams they can fill in to classify objects.

Using visual aids to explain the meaning of each programming block being discussed or used.

Providing pupils with an AI process to practise producing mathematical language:

1. Writing a clear and defined AI prompt:

- “Is the following answer of mine to a mathematics problem sound? The lengths of the sides of the triangle are 2 cm, 3 cm, and 2 cm, which makes it isosceles.”

2. Reading the generated text:

- “Your answer is correct, but you could provide more context by writing ‘which makes it an isosceles triangle’, ensuring the reader fully understands the context without ambiguity.”

3. Comparing the terminology in the AI-generated text with the terminology used in class, e.g. an isosceles triangle, context, ambiguity.

4. Comparing the notations in the AI-generated text with the terminology used in class.

5. Asking for clarification if e.g. a word is not familiar: “Could you tell me what ambiguity means? I am in 3rd grade.”

C2 Numbers and operations

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C2 for grades 1–2, and C2 for grades 7–9.

Content outcomes

C2 Numbers and operations

The pupils

- reinforce their understanding of the decimal system.
- examine and classify numbers, with the goal of diversifying their perception of the structure, the divisibility, and interconnectedness of numbers.
- improve their skills in performing basic mental arithmetic operations.
- practise addition and subtraction algorithms, and their skills in these operations are ensured.
- reinforce their understanding of the concept of multiplication, learn the multiplication tables for numbers 6–9.
- reinforce their mastery of the multiplication tables for numbers 1–10.
- practise the multiplication algorithm, and their mastery of this operation is ensured.
- learn division in cases of both quotient and partition, and practise division by number units.
- utilise the properties of operations and the connections between them.
- are guided to round up figures and to calculate with approximate values, through which they learn to estimate the order of magnitude of the result.
- practise using all four number operations in versatile contexts and using versatile tools.
- get introduced to the concept of negative numbers, and the number area is expanded to negative integers.
- learn the concept of fractions and practise basic arithmetic operations on fractions in various situations.
- practise multiplication and division on natural numbers.
- familiarise themselves with decimal numbers as part of the decimal system and practise basic arithmetic operations with decimal numbers.
- begin to form an understanding of the concept of percentages. A foundation is laid for understanding percentages and percentage values, and the pupils practise calculating them in simple examples.
- get introduced to utilising the connections between fractions, decimal numbers and percentages.

Language outcomes

The pupils practise:

- identifying the operations needed to be carried out in oral tasks in Finnish and in English.
- identifying the operations needed to be carried out in written word-based tasks in Finnish and in English.
- identifying and naming numbers and operations in varying contexts, translating between natural language and mathematical notations, following both Finnish and English conventions
 - e.g. 9 being nine
 - e.g. 13 000 being thirteen thousand following Finnish conventions.
 - e.g. 1,200 being one thousand two hundred following English conventions, and one and two hundred thousandths following Finnish conventions.
 - e.g. a number being *luku* and a digit being *numero*.
 - e.g. the digits (*numerot*) of the number (*luku*) 1,290 being 1, 2, 9, and 0.
 - e.g. nine representing the hundredths in 13,490 when following Finnish conventions, and the tens when following English conventions.
 - e.g. 1,002,000,000,000 being *one trillion two billion* in English but *biljoona kaksi miljardia* in Finnish
 - e.g. $\frac{3}{4}$ being three-fourths, three out of four, three divided by four, and the quotient of three and four.
 - e.g. the sum of 6 and 3 being three added to six and as such, equal to $6 + 3 = 9$.
 - e.g. 5^2 being five to the power of 2 and being the square of 5.
 - e.g. -2 being negative two and $-2 - 3$ being negative two minus three.
- using the verbs *to count*, *to calculate*, and *to find* to fit the context
 - e.g. to count the apples, to calculate the sum in euros, to find the missing number
- comparing different approaches to an assignment, task or problem, and evaluating, assessing, and reasoning which one best suits the scenario, using correct terminology and vocabulary.

- presenting their answers so they can be understood by pupils and teachers, and supporting their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
 - e.g. $6 / 3$ being 2, as there are three twos in six.
- giving constructive feedback on a pupil's, a teacher's or an AI's thought process, and comparing it to their own approach.
- evaluating, assessing, and reasoning if the value they get fits the assignment, task or problem.

Attachments and links

Operations on numbers, by JanneRytkönen

Make a Ten, by PhET Interactive Simulations, University of Colorado Boulder, [Creative Commons CC-BY 4.0](#)

Hundreds, tens, and ones, by Khan Academy, [Creative Commons CC-BY-NC-SA 3.0](#)

Unit: Negative numbers, by Khan Academy, [Creative Commons CC-BY-NC-SA 3.0](#)

Unit: *Absolute value*, by Khan Academy, [Creative Commons CC-BY-NC-SA 3.0](#)

Key words

Operations (Laskutoimitukset)

to count (laskea / luetella), to calculate (laskea laskutoimitus), to find (selvittää), to carry out (an operation) (laskea laskutoimitus)

to add (laskea yhteen), addition (yhteenlasku), a sum (summa), plus (plus)

to subtract (vähentää), subtraction (vähennyslasku), sub- (under), to track (vetää), difference (erotus), to take away (poistaa / ottaa pois), minus (miinus)

to multiply (kertoa), multiplication (kertolasku), times (kertaa), multiplied by (kerrottuna jollakin), multi- (moni-), a product (tulo), a multiplier (kertoja), a multiplicand (kerrottava), to carry over (merkitä muistiin), a factor (tekijä), a prime factor (alkutekijä)

double (kaksinkertainen), twice as many/much as (kaksi kertaa niin monta / paljon kuin), triple (kolminkertainen), three times as many/much as (kolme kertaa niin monta / paljon kuin)

a divisor (jakaja), a dividend (jaettava), a remainder (jakojäännös), a quotient (osamäärä)

the order of operations (laskujärjestys), brackets / round brackets / parentheses () (kaarisulut), square brackets / brackets [] (hakasulut), curly brackets / braces {} (aaltosulut)

Numbers and digits (Luvut ja numerot)

a number (luku), a digit (numero), two-digit (kaksinumeroinen), ones (ykköset) / tens (kymmenet) / hundreds (sadat) / ... / tenths (kymmenesosat) / hundredths (sadasosat) / ...

ten thousand (kymmenen tuhatta) / hundred thousand (sata tuhatta)

to round (pyöristää), to round up (pyöristää ylös), to round down (pyöristää alas), to round to the tens (pyöristää kymmenen tarkkuudelle) / hundreds (pyöristää satojen tarkkuudelle) / ... / tenths (pyöristää kymmenesosien tarkkuudelle) / hundredths (pyöristää sadasosien tarkkuudelle) / ...

to add / subtract / multiply in columns (laskea yhteenlasku / vähennyslasku / kertolasku sarakeittain)

a fraction (murtoluku), one-half (yksi kahdesosa) / one-third (yksi kolmasosa) / one-fourth (yksi neljäsosa) / ..., the numerator (osoittaja), the denominator (nimittäjä), a common denominator (yhteinen nimittäjä), least common multiple lcm (pienin yhteinen jaettava pyj), to expand (laentaa), to reduce (supistaa), a mixed number (sekaluku), a proper fraction (aito murtoluku), an improper fraction (epämurtoluku), the wholes (kokonaiset), the fractional part (murto-osa), a repeating decimal representation (toistuva desimaaliesitys), a fraction of time / length (ajan / pituuden murto-osa)

a percentage (prosenttiosuus), percent (prosentti), to reduce (supistaa), to increase (kasvattaa / kasvaa), to raise (kasvattaa), a discount (alennus)

Ordinal numbers (Järjestysluvut)

first (ensimmäinen), second (toinen), third (kolmas), fourth (neljäs), ...

Classifying and comparing numbers (Lukujen luokittelu ja vertailu)

the number line (lukusuora), a dash (viiva lukusuoralla), more (enemmän), less (vähemmän), fewer (vähemmän), greater than (suurempi kuin), less than (pienempi kuin), (the) most (eniten), (the) least (vähiten), left over (yli jäänyt), equal to (yhtä suuri kuin), to equal (olla yhtä suuri kuin)

a multiplication table (kertotaulu), number bonds (hajotusparit), a fact family (käänteiset laskutoimitusperheet), even (parillinen), odd (pariton), a group of (joukko jotakin)

in total (yhteensä), altogether (yhteensä), lots of (paljon jotakin), a lot of (paljon jotakin)

to reason (päätellä / järkeillä), to figure out (päätellä), to show work (kirjoittaa välivaiheet)

a natural number (luonnollinen luku), a prime number (alkuluku), an integer (kokonaisluku), a positive integer (positiivinen kokonaisluku), a negative integer (negatiivinen kokonaisluku), a power (potenssi), an index (indeksi), a base (kantaluksi), an exponent (eksponentti), a value of (a power / an expression) ((potenssin / lausekkeen) arvo), a square (neliö), squared (toiseen), a cube (kuutio), cubed (kolmanteen)

Scaffolding

Input scaffolding

Revisiting concepts, terminology, vocabulary, and notations regularly, like reinforcing the understanding of fractions being quotients.

Keeping advanced numerals, like four-digit numbers and fractions, and related terminology and vocabulary, like decimal place names, visible in the classroom throughout.

Output scaffolding

Keeping the order of operations, together with the names of each operation, visible in the classroom throughout.

Offering students with sentence diagrams they can fill in to read strings of operations using correct mathematical terminology.

Modeling solving strategies for word-based tasks, e.g. colouring in sentences, creating illustrations, rewording and rephrasing (rf. to output scaffolding methods in C3 and C5 for grades 7–9).

C3 Algebra

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entry: C3 for grades 7–9.

Content outcomes

C3 Algebra

The pupils

- observe the regularities of number sequences and continue a number sequence following its rule.
- get to know the concept of the unknown.
- examine equations and solve them by reasoning and experimentation.

Language outcomes

The pupils practise:

- describing and determining rules to explain repeating patterns
 - e.g. the sequence 2, 4, 6, 8, ... consisting of even numbers or of every other number, starting with 2.
 - e.g. the sequence $-4, -1, 2, 5, \dots$ where each member increases by three at each step.
 - e.g. the sequence 0,1; 0,01; 0,001; ... having members get an additional zero decimal or having members divided by 10.
- reading expressions and equations involving unknowns
 - e.g. $4 + 2x$ as “four plus two x’s” or “as two x’s added to four”.
 - e.g. $4 + 2x = 6$ as “two x’s added to four equals six”.
- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- giving constructive feedback on a pupil’s, a teacher’s or an AI’s thought process, and comparing it to their own approach
- presenting their answers so they can be understood by pupils and teachers, and supporting their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- evaluating, assessing, and reasoning whether the obtained value fits the assignment, task or problem.
 - e.g. the number of objects being a natural number unless stated otherwise.

Attachments and links

Patterns with numbers, by Khan Academy,
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Key words

a (number) sequence ((luku)jono), a pattern (kuvio / toistuva ominaisuus), to repeat (toistua / toistaa), repetitive (toistuva), repetition (toisto), to fill out (täydentää), an element / a member (jäsen), the first / second / third / ... element / member (ensimmäinen / toinen / kolmas / ... jäsen)

to be equal to (olla yhtä suuri kuin), to equal (olla yhtä suuri kuin), an equation (yhtälö), an unknown (tuntematon), a solution (ratkaisu)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping examples of sequence patterns visible in the classroom, e.g. adding at each step, getting divided at each step,

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Providing students with an AI process to practise producing mathematical language:

1. Writing a clear and defined AI prompt:

- “I am a Finnish fifth-grader. I am tasked with describing the pattern of the sequence 0,1; 0,01; 0,001; 0,000 1; I only know how to describe it in Finnish (pilkku siirtyy joka askeleella kerran vasemmalle). How would I say the same thing in English?”

2. Reading the generated text:

- “In English, you can describe the pattern as follows: “The comma shifts one place to the left with each step.””

3. Comparing the terminology in the AI-generated text with the terminology used in class, e.g. a comma, to shift, a step.

4. Comparing the notations in the AI-generated text with the terminology used in class, e.g. using commas or points as decimal separators.

5. Asking for clarification if e.g. a word is not familiar:

- “What do you mean by comma? I am a fifth-grader and have not seen that word before.”

C4 Geometry

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C3 for grades 1–2, and C4 and C5 for grades 7–9.

Content outcomes

C4 Geometry and measuring

The pupils

- build, draw, examine, and classify objects and figures.
- classify objects into cylinders, cones and other objects.
- familiarise themselves more closely with rectangular prisms, round cylinders, circular cones and pyramids.
- classify plane figures into polygons and other figures, and examine their properties.
- learn more about triangles, quadrangles, and circles.
- familiarise themselves with the concepts of a point, a segment of a line, a straight angle, and an angle.
- practise drawing, measuring, and classifying angles.
- study symmetry in proportion to a line.
- guided to notice rotational and translational symmetries in their surroundings, for example in art.
- are first introduced to the first quarter of the system of coordinates and later to all quarters.
- are familiarised with the concept of scale, which is applied to enlargements and reductions.
- are instructed in utilising the concept of scale when working with maps.
- practise measuring and pay attention to the accuracy of measurement, estimation of the measurement results, and verifying measurements.
- measure and calculate the circumferences and surface areas of figures of different shapes and the volumes of rectangular prisms.
- are guided to understand how the system of measurement units is structured.
- practise making unit conversions with the most common units of measurement.

Language outcomes

Listening skills

The pupils practise:

- identifying terminology and vocabulary related to geometry and measuring from spoken Finnish and English, and acting according to detailed oral instructions of varying complexity.
 - e.g. “Draw a circle whose radius is 3 centimetres.”
 - e.g. “You will need your protractor (in the next assignment), in order to draw an (isosceles obtuse) triangle (whose area is ten centimetres squared).”

Reading skills

The pupils practise:

- identifying mathematical terminology and vocabulary in written Finnish and English, and acting according to detailed written instruction.
- identifying parts of shapes and figures by their written names both in Finnish and in English.
 - e.g. “The base of this figure is a circle, and there is a right angle there, so this must be a right circular cone.”
 - e.g. V representing **v**olume, p representing **p**erimeter, h representing **h**eight, A representing **a**rea (rf. Attachments and links).

Speaking, writing, and illustrating skills

The pupils practise:

- classifying geometric shapes and figures and explaining their key properties of using their own words
 - e.g. “A cuboid has six faces, twelve edges, and eight vertices.”
 - e.g. “This trapezoid can be broken down into a rectangle and two right triangles.”
- explaining their thought process when carrying out measurements.
 - e.g. “I placed the middle part of the protractor here (at the vertex) and then read the degrees (over here).”
- rephrasing assignments in their own words.
 - e.g. “To build this (right circular cylinder) out of paper, I need to cut out two (congruent, identical) circles and a rectangle.”

- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
 - e.g. “Instead of searching for a direct solution, we should try calculating the area of this trapezoid by calculating the area of these two triangles and of this rectangle, and then adding them up.”
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- giving constructive feedback on a pupil’s, a teacher’s or an AI’s thought process, and comparing it to their own approach.
- evaluating, assessing, and reasoning whether the obtained value fits the assignment, task or problem.
 - e.g. “As the triangle is inside the square, its area should not be greater than the area of the square.”

Attachments and links

[Classifying quadrilaterals](#)

[Classifying triangles](#)

[Calculations on squares](#)

[Calculations on rectangles](#)

[Calculations on parallelograms](#)

[Calculations on triangles](#)

[Geogebra assignments on planar geometry](#)

A table of geometrical vocabulary in both Finnish and English with explanations.

suomeksi	in English	explanation in mathematics	explanation in layman's terms
piste	point	An object with no width, height, or depth.	Drawn as a dot with a pen.
jana	line segment	An object with length only, straight, with both a starting point and an end point.	Drawn with a ruler, a dot or a dash on both ends.
puolisuora	ray	An object with length only, straight, with a starting point but no end point.	Drawn with a ruler, a dot or a dash on one end.
suora	line	An object with length only, with no starting or end point.	Drawn with a ruler.
viiva	curve	An object with length only, with no starting or end point.	Drawn freehand.
murtoviiva	polygonal chain	An object consisting of line segments, whose end points are starting points for consequent line segments.	Drawn with a ruler, by rotating it in between drawing the line segments in succession.
taso	plane	An object with width and depth, flat.	The page in your notebook.

Table of geometrical abbreviation with full names in Finnish and English.

abbreviation	original	Finnish translation
V	volume	tilavuus
A	area	pinta-ala
h	height	korkeus
p	perimeter	piiri
d	diameter	halkaisija

abbreviation	original	Finnish translation
r	radius	säde
a, b, c	Used to denote side length.	
x, y, z	Used to mark denote side length.	

Key words

Geometry (Geometria)

a feature (ominaisuus / piirre / yksityiskohta),
a characteristic (ominaisuus/määrittävä ominaisuus),
a property (ominaisuus)

to be symmetrical to each other / one another (olla symmetrinen jonkin / joidenkin suhteen), symmetrical shapes (symmetriset kuviot), congruence (yhtenevyys), to be congruent to (olla yhtenevä jonkin kanssa), similarity (yhdenmuotoisuus), to be similar to (olla yhdenmuotoinen jonkin kanssa), scale e.g. one to one hundred thousand 1 : 100,000 (mittakaava, esim. yhden suhde sataan tuhanteen 1 : 100 000), to enlarge (suurentaa), enlarged (suurennettu), to reduce (pienetää), reduced (pienennetty), a ratio (suhde)

a circle (ympyrä), circular (ympyrän muotoinen), a radius (säde), a diameter (halkaisija), a circumference (kehän pituus), a centre / center (point) (keskipiste), a chord (jänne), a sector (sektori), an arc (kaari)

a cylinder (lieriö), cylindrical (lieriön muotoinen), a cone (kartio), conic (kartio-), a base (pohja), a face (tahko), an edge (särmä), a vertex (kärki), a point (piste), a circular cylinder / a circle-based cylinder (ympyräpohjainen lieriö), a circular cone / a circle-based cone (ympyräpohjainen kartio), a cuboid (kuusitahokas), right (suora), oblique (vino), a prism (särmio)

a line (suora), a line segment (jana), a polygon (monikulmio), a polygonal chain (murtoviiva), a ray (puolisuora), to be parallel to (olla yhdensuuntainen jonkin kanssa), to be perpendicular to (olla kohtisuorassa jotakin vastaan)

an angle (kulma), an acute angle (terävä kulma), a right angle (suora kulma), an obtuse angle (tylppä kulma), a convex angle (kovera kulma), a concave angle (kupera kulma); an acute triangle (teräväkulmainen kolmio), a right triangle (suorakulmainen kolmio), an obtuse triangle (tylppäkulmainen kolmio), an oblique triangle (kolmio, jossa ei ole suoraa kulmaa), an isosceles triangle (tasakylkinen kolmio), an equilateral triangle (tasasivuinen kolmio), a scalene triangle (epäsäännöllinen kolmio); a straight angle (oikokulma), a protractor (kolmiovivain / geokolmio), a degree (aste)

a parallelogram (suunnikas), a rhombus (many rhombi) (neljäkäs)

a diagonal (lävistäjä)

a coordinate plane (tasokoordinaatisto), a coordinate (koordinaatti), the 1st / I quadrant (1. / I neljännes), the origin (origo), the x -axis (x -akseli), the y -axis (y -akseli), the 2nd / II quadrant (2. / II neljännes), the 3rd / III quadrant (3. / III neljännes), the 4th / IV quadrant (4. / IV neljännes), an ordered pair of numbers (järjestetty lukupari)

a compass (kompassi), North N (pohjoinen), East E (itä), South S (etelä), West W (länsi), North-East NE (koillinen), North-West NW (lounas), South-East SE (kaakko), South-West SW (lounas)

Measuring (Mittaaminen)

a perimeter (piiri), an area (pinta-ala), a surface area (pinnan pinta-ala), a square metre / a metre squared (neliömetri), a hectare (hehtaari), an are (aari), an acre (eekkeri)

mass (massa)

Scaffolding

Input scaffolding

Keeping shapes, their names, and key characteristics visible in the classroom throughout.

Comparing new geometric shapes to ones learned previously, by creating a glossary chapter-by-chapter with layman-type vocabulary the students feel at home with (rf. Attachments and link of this section for an example).

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

C5 Data processing and software, statistics, and probability

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C4 for grades 1–2, and C6 for grades 7–9.

Content outcomes

C5 Data processing and software, statistics, and probability

The pupils

- develop their skills in systematically collecting data on topics that are of interest to them.
- record and present data in the form of tables and diagrams.
- are introduced, of the statistical key figures, to the greatest and least value, the average, and the mode.
- familiarise themselves with probability in everyday situations by concluding whether an event is impossible, possible, or certain.

Language outcomes

The pupils practise:

- classifying events as impossible, possible or certain, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- reading and finding the average, mode, median, and the greatest and least value from data in varying forms.
 - e.g. in tables, maps, charts, diagrams, and graphs.
- forming clear and unambiguous data collection questions to ensure consistent responses.
- using concise keywords when searching for data on a given topic.
- assessing the reliability of the data source.
- assessing which representations best suit the given data source and the assignment.
 - e.g. a table, a map, a chart, a diagram or a graph.
- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- giving constructive feedback on a pupil's, a teacher's or an AI's thought process, and comparing it to their own approach
- evaluating, assessing, and reasoning whether the obtained value fits the assignment, task or problem.
 - e.g. an average or a median being in the middle of the range for the data.
 - e.g. the maximum being the greatest of the numbers involved.

Attachments and links

[Different graph types with context & Add a chart or graph](#), on Google Docs Help Center

[Different graph types with context & Add a chart or graph](#), on Microsoft Support

Key words

a table (taulukko), a column (sarake), a row (rivi), a cell (solu), to plot (piirtää kuvaaja / kaavio / diagrammi), a line graph (viivakaavio), data (tieto / data), a data set (tietue / datajoukko), a data group (tietue / datajoukko), a bar graph (sarakekaavio), a pictogram (havainnollistava kuvake), a pie chart (piirakkakaavio)

a set (joukko), to intersect (leikata), an intersection (leikkaus), to be disjoint (olla erilliset), a subset (osajoukko), a Venn diagram (Venn-diagrammi), a tree diagram (puukaavio)

mean (keskiarvo), range (vaihteluväli), greatest (suurin), least (pienin), maximum (maksimi), minimum (minimi), mode (moodi), average (keskiarvo), median (mediaani)

a probability (todennäköisyys), probable (todennäköinen), equally probable (yhtä todennäköinen), impossible (mahdoton), possible (mahdollinen), certain (varma)

a form / poll / questionnaire / query (kysely), a question (kysymys), a response (vastaus), a respondent (vastaaja), a search word (hakusana)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Offering students with sentence diagrams they can fill in to interpret graphs and read data in table format.

7th–9th grade

C1 Thinking skills and methods

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C1 for grades 1–2, and C1 for grades 3–6.

Content outcomes

C1 Thinking skills and methods

The students practise

- activities requiring logical thinking such as discovering rules and dependencies and presenting them accurately.
- considering and determining the number of possible alternatives.
- interpreting and producing mathematical notations.
- providing proof.
- determining the truth value of propositions.

The students

- deepen their algorithmic thinking.
- strengthen their reasoning and argumentation skills.
- programme while learning good programming practices.
- use their own or ready-made computer programmes as a part of learning mathematics.

Language outcomes

Listening and reading skills

The students practise:

- understanding oral instructions in spoken Finnish and in English, and acting accordingly.
- identifying the key information in orally-given word-based tasks in Finnish and in English.

The students practise:

- solving oral and written problems, tasks, and assignments by:
 - identifying the context of the task based on its notations and mathematical terminology.
 - specifying the goal of the task.
 - predicting a range and the number for possible solutions for the task.
 - reflecting if a solution is appropriate for the task setting.

The students practise:

- translating blocks of code into natural language commands, and vice versa.

Speaking and writing skills

The students practise:

- expressing themselves orally and in written form by, for example:
 - asking questions relevant to the topic, task or assignment.
 - explaining their thought process behind their solution to a problem concisely and by using relevant and correct terminology and vocabulary.
- supporting their reasoning or solution with concise, relevant arguments, clearly written intermediate steps, and appropriate, understandable figures.

The students practise:

- using correct terminology and vocabulary when reading mathematical symbols.

The students practise:

- describing the structure of code in a programming environment.
- explaining the purpose of blocks of code in a programming environment.
- elaborating on whether or not a program follows good programming practises.
- elaborating on why a specific computer program suits a task.

The students practise:

- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- giving constructive feedback on a pupil's, a teacher's or an AI's thought process, and comparing it to their own approach
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- evaluating, assessing, and reasoning whether the obtained value is appropriate for the assignment, task or problem.

Attachments and links

What does the task expect me to do?



Key words

Classifying and comparing numbers (Lukujen luokittelu ja vertailu)

to group (ryhmitellä), to sort (järjestää), to compare (verrata / vertailla); to be the same as (olla sama kuin), similar to (samankaltainen kuin), different from (eri kuin / erilainen kuin), to belong in (kuulua johonkin); greater than (suurempi kuin), less than (pienempi kuin), equal to (yhtä suuri kuin), to equal (olla yhtä suuri kuin)

Thinking processes (Ajatteluketjut)

a solution (ratkaisu), to solve (ratkaista), a reasoning (perustelu), to reason (perustella / järkeillä), a proof (todistus / osoitus), to prove (todistaa / osoittaa), to find (selvittää), to show one's work (kirjoittaa välivaiheet), a proof (todistus / osoitus), to explain (selittää), an explanation (selitys), to justify (perustella), a justification (perustelu), to be just (olla perusteltu), true (tosi), false (epätosi), a true statement (tosi väittämä), a false statement (epätosi väittämä), to be plausible (olla mahdollinen)

a pattern (kuvio / toistuva ominaisuus), a sequence (jono), colo(u)r (väri), size (koko)

Programming (Ohjelmointi)

a program (ohjelma), to program (ohjelmoida), a block of code (koodin pätkä), to code (ohjelmoida / koodata), a step (askel), forward (eteenpäin), backwards (taaksepäin), (a set of) instructions (ohjeet), to estimate (arvioida), to debug (etsiä ja korjata virhe / bugi), a bug (bugi), an algorithm (algoritmi), a sprite (hahmon kuvake), a command (käsky), to command (käskeä), an input (syöte), an output (tuloste); if–then–else (jos–niin–muutoin), until (kunnes), repetition (toisto), repetitive (toistuva / toistava), a loop (silmukka); a variable (muuttuja), a selection (valinta), to select (valita)

a function (funktio / toiminto), a range (arvojoukko), an argument (argumentti / annettava arvo), to select a range of cells/rows/columns (valita tietyt solut/rivit/sarakkeet), to auto-fill (täyttää automaattisesti)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping adopted concepts, terminology, vocabulary, and notations visible in the classroom, while introducing new visual aids alongside related ones.

Using AI to transform data into convenient forms:

- E.g. to form a **row-by-row handout** based on the terminology lists in this document:
 - “Could you reformat the following list so that all the terms are on their own separate rows?”
- E.g. to form a **row-by-row handout** based on the terminology lists in this document in **either language only**:
 - “Could you reformat the following list so that all the terms are on their own separate rows and that only the words in Finnish are included?”

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Offering students with sentence diagrams they can fill in to classify objects.

Providing visual aids to explain the meaning of each programming block being discussed or used.

Providing students with an AI process to practise producing mathematical language:

1. Writing a clear and defined AI prompt:

“Could you explain with steps shown which one is greater: two-thirds or three-fourths? I am in 8th grade, and we use Finnish mathematical notations.”

2. Reading the generated text:

- “Sure! To compare two-thirds and three-fourths, we can find a common denominator and then compare their numerators.
- Convert two-thirds to have a denominator of 12: $2/3 \times 4/4 = 8/12$
- Convert three-fourths to have a denominator of 12: $3/4 \times 3/3 = 9/12$
- Now, we can see that $9/12$ is greater than $8/12$, so three-fourths ($9/12$) is greater than two-thirds ($8/12$).”

3. **Comparing the terminology** in the AI-generated text with the terminology used in class, e.g. denominator, numerator.

4. **Comparing the notations** in the AI-generated text with the terminology used in class, e.g. using a cross \times to denote multiplication.

5. **Asking for clarification** if e.g. a word is not familiar: “What are denominators? I am an eight-grader.”

C2 Numbers and operations

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C2 for grades 1–2, and C2 for grades 3–6.

Content outcomes

C2 Numbers and operations

The students

- practise basic arithmetic operations also with negative numbers.
- strengthen their arithmetic skills using fractions
- learn to multiply and divide by fractions.
- familiarise themselves with the concepts of opposite numbers, reciprocal numbers, and absolute values.
- expand their concept of numbers to real numbers.
- familiarise themselves with divisibility and divide numbers into prime factors.
- enhance their proficiency in performing operations on decimal fractions.
- strengthen their understanding of the difference between exact values and approximations and rounding the results of operations.
- reinforce their understanding of the concept of percentages.
- practise calculating percentages and calculating the amount a percentage expresses of a whole, and also learn to calculate a changed value, a basic value, and percentage of change and comparison.
- practise calculating exponentials using whole-number exponents.
- familiarise themselves with the concept of the square root and use the square root in operations.

Language outcomes

The students practise:

- identifying key mathematical terminology and vocabulary in written Finnish and English.
- identifying the key information in written word-based tasks in Finnish and in English.
- rephrasing assignments in their own words.
- translating numerals into numbers and vice versa, while following natural language conventions
 - e.g. 🍎🍎🍎 being equal to 3
 - e.g. 1,200 being one thousand two hundred following English conventions, and one and two hundred thousandths following Finnish conventions.
 - e.g. the digits (*numerot*) of the number (*luku*) 1,290 being 1, 2, 9, and 0.
 - e.g. nine representing the hundredths in 13,490 when following Finnish conventions, and the tens when following English conventions.
- reading and writing numbers in natural language and vice versa
 - e.g. 9 being nine and 13 being **thirteen**.
 - e.g. $\frac{3}{4}$ being three-fourths.
 - e.g. -2 being negative two and $-2 - 3$ being negative two minus three.
- using the verbs *to count*, *to calculate*, and *to find*, as appropriate for the context.
 - e.g. to count the apples, to calculate the sum in euros, to find the missing number
- converting operations on numbers to and from natural language using appropriate terminology
 - e.g. $2,4 \cdot 35$ being multiplication: being two-point-four times thirty-five, being two-point-four multiplied by thirty-five, being the product of 2,4 and 35 and, furthermore, being the product of 24 and 35 divided by 10.
 - e.g. 5^2 being five to the power of two, being the square of 5, and being the second power of 5
- explaining their thought process when carrying out operations.

The students practise:

- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- giving constructive feedback on a pupil's, a teacher's or an AI's thought process, and comparing it to their own approach
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- evaluating, assessing, and reasoning whether the obtained value is appropriate for the assignment, task or problem.

Attachments and links

[Reading operations](#) – Tasks regarding how numbers and operations are read, by JanneRytönen

- Please observe that the link is only functional if you are logged in onto Blooket before clicking on it.

[Hundreds, tens, and ones](#), by Khan Academy, [Creative Commons CC-BY-NC-SA 3.0](#)

Unit: [Absolute value](#), by Khan Academy, [Creative Commons CC-BY-NC-SA 3.0](#)

Unit: [Negative numbers](#), by Khan Academy, [Creative Commons CC-BY-NC-SA 3.0](#)

Speaking frame AB exercise:

[Reading numbers and operations](#)

Speaking frame: Reading numbers and operations ([printable](#))

Key words

Operations

to count (laskea / luetella), to calculate (laskea laskutoimitus), to find (selvittää / ratkaista)

to add (laskea yhteen), to be added to (laskea yhteen johonkin), addition (yhteenlasku), sum (summa), to subtract (vähentää), to be subtracted from (vähentää jostakin), subtraction (vähennyslasku), difference (erotus), to take away (poistaa jostakin / ottaa jostakin pois), plus (plus), minus (miinus), positive (positiivinen), negative (negatiivinen)

to multiply (kertoa), multiplication (kertolasku), times (kertaa), to be multiplied by (kerrottuna jollakin), multiplier (kertoja), product (tulo), factor (tulontekijä), factorise (jakaa tekijöihin), to divide (jakaa), to be divided by (jaettuna jollakin), division (jakolasku), quotient (osamäärä), to fit into (mahtua johonkin), to consist of (koostua jostakin)

the order of operations (laskujärjestys), brackets / round brackets / parentheses () (kaarisulut), square brackets / brackets [] (hakasulut), curly brackets / braces {} (aallosulut)

to carry out a root (laskea neliöjuuren arvon), to carry out a power (laskea potenssin arvo)

Numbers and digits (Luvut ja numerot)

a number (luku), a digit (numero), a place value (paikkamerkintä), the units/ones (ykköset), the tens (kymmenet), the hundreds (sadat), the thousands (tuhannet), the ten thousands (kymmenet tuhannet), the hundred thousands (sadat tuhannet), the millions (miljoonat), the billions / milliards (miljardit), the trillions / billions (biljoonat), ..., the tenths (kymmenesosat), the hundredths (sadasosat), the thousandths (tuhannesosat), ..., the decimal separator (desimaalierotin), a sign (etumerkki)

Structuring numbers (Lukujen rakenne)

the number line (lukusuora), a dash (viiva lukusuoralla), more (enemmän), less (vähemmän), fewer (vähemmän), greater than (enemmän kuin), less than (vähemmän kuin), (the) most (eniten), (the) least (vähiten), left over (yli jäänyt), equal to (yhtä suuri kuin), to equal (olla yhtä suuri kuin)

the opposite number of / the additive inverse of (jonkin vastaluku), the reciprocal number of / the multiplicative inverse of (jonkin käänteisluku), the absolute value of (jonkin itseisarvo), equidistant from (yhtä kaukana jostakin)

divisibility (jaollisuus), to be divisible by (olla jaollinen jollakin), a divisor (jakaja), a factor (tekijä), a prime factor (alkutekijä), a prime number (alkuluku), a composite number (yhdistetty luku), a factorisation (tekijähajotelma), the prime number composition of (jonkin alkutekijähajotelma)

a cipher (salausmenetelmä), Caesar cipher (Caesar-salaus), to encrypt (salata), encryption (salaus), to decrypt (purkaa salaus), decryption (salauksen purkaminen), a brute force attack (väsytyshyökkäys), to brute force (tehdä väsytyshyökkäys)

Fractions (Murtoluvut)

a fraction (murtoluku), one-half (yksi kahdesosa) / one-third (yksi kolmasosa) / one-fourth (yksi neljäsosa) / ... / two-thirds (kaksi kolmasosaa) / ..., the numerator (osoittaja), the denominator (nimittäjä), a common denominator (yhteinen nimittäjä), to expand (laventaa), to reduce / simplify (supistaa), to simplify (sieventää), a mixed number (sekaluku), a fraction (murtolukumuoto), a proper fraction (aito murtoluku), an improper fraction (epämurtoluku), the wholes (kokonaiset), the fractional part (murto-osa), a repeating decimal representation (päättymätön jaksollinen desimaaliluku)

Powers and exponentiation (Potenssit)

exponentiation (potenssiin korottaminen), a power (potenssi), a base (kantaluku), an exponent (eksponentti), to raise a number to a power (korottaa luku potenssiin), x raised to the power of n / x to the n th power / the n th power of x (x potenssiin n / x korotettuna n . potenssiin), the square of (jonkin neliö), the cube of (jonkin kuutio), a root of (jonkin juuri), the square root of (jonkin neliöjuuri), a radical sign / a radical symbol / a radix / a surd (juurimerkki)

Number sets (Lukujoukot)

to be natural number (olla luonnollinen luku); an integer (kokonaisluku), a positive number (positiivinen luku), a nonnegative number (ei-negatiivinen luku), a negative number (negatiivinen luku), a nonpositive number (ei-positiivinen luku); a rational number (rationaaliluku), an irrational number (irrationaaliluku); a real number (reaaliluku); to round to (pyöristää johonkin), a significant digit (merkitsevä numero), a decimal place/ n decimal places (desimaalien tarkkuudelle / n desimaalin tarkkuudelle); an exact value (tarkka arvo), an approximate value (liikiarvo), an approximation (liikiarvo), to approximate (muodostaa liikiarvo), an estimated value (arvioitu arvo), an estimate (arvioitu arvo), to estimate (arvioida arvo)

Percentages (Prosenttilaskenta)

a percentage (prosenttiosuus), x percent of (x prosenttia jostakin), a percentage coefficient (prosenttikerroin), the percentage of change (muutosprosentti), the percentage of comparison (vertailuprosentti), the basic value (perusarvo); a percentage point (prosenttiyksikkö); a unit (yksikkö); a mass percentage (massaprosenttiosuus), mass percent (massaprosentti), a volume percentage (tilavuusprosenttiosuus), volume percent (tilavuusprosentti), a solution (liuos); a permillage (promilleosuus), x permille of (x promillea jostakin); a tax (vero), a tax card (verokortti), a tax rate (veroprocentti), a withholding rate (ennakonpidätysprosentti), income (tulo), pay (maksu), benefit (etuus), pension (eläke), annual (vuosittainen), state income tax (valtion tulovero), municipal tax (kunnallisvero), church tax (kirkollisvero), health insurance contribution (sairausvakuutusmaksu), wage-related payments (veroluonteiset maksut); interest (korko), an interest rate (korkokanta), capital (pääoma), stocks (osakkeet), a fund (rahasto), to invest (sijoittaa), inflation (inflaatio), a loan (laina), credit (luotto), p.a. per annum (vuodessa)

Scaffolding

Input scaffolding

Revisiting concepts, terminology, vocabulary, and notations regularly, like reinforcing the understanding of fractions being quotients.

Keeping advanced numerals, like four-digit numbers and fractions, and related terminology and vocabulary, like decimal place names, visible in the classroom throughout.

Output scaffolding

Keeping the order of operations, together with the names of each operation, visible in the classroom throughout.

Reading operations Laskutoimitusten lukeminen

- ① Find the innermost set of brackets.

Etsi sisimmät sulkeet.

() parentheses	() kaarisulkeet
[] square brackets	[] hakasulkeet
{ } curly brackets	{ } aaltosulkeet

- ② Carry out powers and roots.

Lasko potenssit ja juuret.

$$2^5$$

two to the power of five
kaksi potenssiin viisi

$$\sqrt{16}$$

the square root of sixteen
luvun kuusitoista nelijöjuuri

③ Multiply and divide, and simplify fractions.

Laske kerto- ja jakolaskut ja sievennä murtoluvut.

$$2 \cdot 6$$

two times six
kaksi kertaa kuusi

$$81 : 7$$

eighty-one divided by seven
kahdeksantymmentäyksi jaettuna seitsemällä

$$\frac{21}{36}$$

twenty-one thirty-sixths
kaksikymmentäyksi kolmekymmentäkuudesosaa

④ Add and subtract.

Laske yhteen- ja vähennyslaskut

$$6,5 + 7,1$$

six and a half plus seven and a tenth
six point five plus seven point one

kuusi ja puoli plus seitsemän ja yksi kymmenesosa
kuusi pilkku viisi plus seitsemän pilkku yksi

$$1 - (-2)$$

one minus negative two
yksi miinus miinus kaksi

$$-3 - (-1) = -2$$

negative three minus negative one
is equal to negative two

negative three minus negative one
equals negative two

miinus kolme miinus miinus yksi
on yhtä suuri kuin miinus kaksi

⑤ Repeat from step ① onwards.

Toista askeleesta ① eteenpäin.

Expanding, reducing, and
converting

$$\frac{2^2}{14} = \frac{1}{7}$$

two-fourteenths is reduced by two
to one-seventh

$$2) \frac{6}{43} = \frac{12}{86}$$

six-forty-thirds is expanded by two
to twelve-eighty-sixths

$$3 \frac{4}{5} = \frac{3 \cdot 5 + 4}{5} = \frac{19}{5}$$

three and four-fifths is converted to nineteen-fifths

Providing students with sentence diagrams to assist them in reading strings of operations using appropriate mathematical terminology throughout.

$$\frac{2}{14} \div \left(\frac{3}{7} + \frac{5}{7} \right)$$

two - fourteenths divided by
the sum of
three - sevenths and five - sevenths

$$\frac{2}{14} \div \frac{3}{7} + \frac{5}{7}$$

two - fourteenths divided by
three - sevenths,
and this is added to five - sevenths

Modeling solving strategies for word-based tasks, e.g. colouring in sentences, creating illustrations, rewording and rephrasing (rf. to output scaffolding methods in C3 and C5 for grades 7–9).

C3 Algebra

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entry: C3 for grades 3–6.

Content outcomes

C3 Algebra

The students

- familiarise themselves with the concept of variables and calculating the value of a mathematical expression.
- practice reducing exponential expressions.
- familiarise themselves with the concept of the polynomial and practise adding, subtracting, and multiplying polynomials.
- practise forming, reducing, and simplifying expressions.
- form and solve first-degree equations and incomplete second-degree equations.
- solve pairs of equations graphically and algebraically.
- familiarise themselves with first-degree inequalities and solve them.
- deepen their skills in examining and forming number sequences.
- use proportions in problem-solving.

Language outcomes

The students practise:

- describing and determining rules to explain repeating patterns
 - e.g. the sequence 2, 4, 6, 8, ... consisting of even numbers or of every other number, starting with 2.
 - e.g. the sequence $-4, -1, 2, 5, \dots$ where each member increases by three at each step.
 - e.g. the sequence 0,1; 0,01; 0,001; ... having members get an additional zero decimal or having members divided by 10.
- reading expressions and equations involving variables
 - e.g. $4 + 2x$ as “two x’s added to four” or “four plus two x’s”.
 - e.g. $4 + 2x = 6$ as “two x’s added to four equals six”.
- reading terminology and vocabulary containing loaned prefixes and suffixes
 - e.g. mono-, bi-, and tri- representing 1, 2, and 3, respectively.

The students practise:

- verbalising mathematical notations in natural language and vice versa.
- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- giving constructive feedback on a pupil’s, a teacher’s or an AI’s thought process, and comparing it to their own approach.
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- evaluating, assessing, and reasoning whether the obtained value is appropriate for the assignment, task or problem.
 - e.g. the number of objects being a natural number unless stated otherwise.

Attachments and links

Unit: *Intro to polynomials*, by Khan Academy,
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Key words

Variables, expressions, and polynomials (Muuttujat, lausekkeet ja polynomit)

a variable (muuttuja), a mathematical expression (matemaattinen lauseke), a value of (jonkin arvo), an operation (laskutoimitus), addition (yhteenlasku), sum (summa), subtraction (vähennyslasku), difference (erotus), multiplication (kertolasku), product (tulo), division (jakolasku), quotient (osamäärä)

the commutative property (vaihdannaisuus), (to be) commutative ((olla) vaihdannainen), the distributive property (osittelulaki), (to be) distributive (noudattaa osittelulakia)

a power (potenssi), a base (kantaluksi), an exponent (eksponentti), a square root (neliöjuuri), scientific notation (kymmenpotenssimuoto)

a monomial (monomi), a binomial (binomi), a trinomial (trinomi), a polynomial (polynomi)

an expression (lauseke), a term (termi), a coefficient (kerroin), a variable (muuttuja), a power of a variable (muuttujan potenssi), a variable part (muuttujaosa), a degree (asteluku), n th degree (n . aste), a constant term (vakiotermi), like terms (samanmuotoiset termit)

to simplify (sieventää), to find (selvittää/ratkaista), to calculate (laskea)

to substitute a for x (sijoittaa luku a luvun x paikalle), a value (arvo)

Exponent rules (Potenssien laskusäännöt)

exponentiation (potenssiin korottaminen), a power (potenssi), a base (kantaluksi), like bases (samankantaiset); an exponent (eksponentti), to raise a number to a power (korottaa luku potenssiin), x raised to the power of n / x to the n th power / the n th power of x (x potenssiin n / luku x korotettuna n . potenssiin)

“The product of powers with like bases equals the base to the sum of the exponents.” (“Samankantaisten potenssien tulo voidaan sieventää potenssiksi, jonka kantaluksi on alkuperäinen kantaluksi ja eksponenttina alkuperäisten eksponenttien summa.”)

“The quotient of powers with like bases equals the base to the difference of the exponents.” (“Samankantaisten potenssien osamäärä voidaan sieventää potenssiksi, jonka kantaluksi on alkuperäinen kantaluksi ja eksponenttina alkuperäisten eksponenttien erotus.”)

“The power of a power equals the base to the product of the exponents.” (“Potenssin potenssi on yhtä suuri kuin kantaluksi potenssiin eksponenttien tulo.”)

“The power of a product equals the product of the powers.” (“Tulon potenssi on yhtä suuri kuin potenssien tulo.”)

“The power of a quotient equals the quotient of the powers.” (“Osamäärän potenssi on yhtä suuri kuin potenssien osamäärä.”)

“A non-zero base to the nil/zeroth power equals 1.” (“Nollasta eroava kantaluksi potenssiin nolla on yhtä suuri kuin 1.”)

scientific notation / scientific form / standard index form / standard form (kymmenpotenssimuoto), in base ten (kymmenjärjestelmässä), the decimal system (desimaalijärjestelmä / kymmenjärjestelmä), a decimal place (kymmenjärjestelmän paikka-arvo), a coefficient (kerroin)

a unit (yksikkö), the metric / SI prefixes / SI prefixes (SI-etuliitteet); nano (nano-), micro (mikro-) μ mu (my), milli (milli-), centi (sentti-), deci (desi-), deca (deka-), hecto (hekto-), kilo (kilo-), mega (mega-), giga (giga-), tera (tera-); the Greek alphabet (kreikkalaiset aakkoset)

Equations (Yhtälöt)

an equation (yhtälö), a variable (muuttuja), variable term (muuttujatermi), constant term (vakiotermi), to be unknown (olla tuntematon), an n th degree equation (n . asteen yhtälö), a solution (ratkaisu), a root (juuri), an identical equation (identtinen yhtälö), to be true (olla totta), to be false (olla epätotta), equal sign (yhtäsuuruusmerkki), operation line (operaatioviiva)

Please observe: there is no specific universally recognized name for the lines used to mark down steps when solving an equation.

“The equation is satisfied by all values of real numbers.” (“Kaikki reaalityluvut toteuttavat yhtälön.”) / “All real numbers are solutions to the equation.” (“Kaikki reaalityluvut ovat yhtälön ratkaisuja.”)

“The equation has no solution.” (“Yhtälöllä ei ole ratkaisua.”) / “No real number is a solution to the equation.” (“Mikään reaalityluku ei toteuta yhtälöä.”) / “The equation is not satisfied by any value of real numbers.” (“Yksikään reaalityluku ei ole yhtälön ratkaisu.”)

Systems of equations (Yhtälöryhmät)

a pair of equations (yhtälöpari), a system of equations (yhtälöryhmä), simultaneous equations (yhtälöpari), a solution (ratkaisu)

to solve graphically (ratkaista graafisesti), a graphical solution (graafinen ratkaisu), an xy -coordinate system (xy -koordinaatisto), an axis (akseli), the x -axis (x -akseli), the y -axis (y -akseli), the origin (origo), the coordinates (koordinaatit), an ordered pair of numbers (järjestetty lukupari), the x -coordinate (x -koordinaatti), the y -coordinate (y -koordinaatti), a point (piste), a line (suora), a graph (kuvaaja), an intersection (leikkauspiste), to intersect (leikata), to be parallel to (olla yhdensuuntainen jonkin kanssa), to coincide (olla sama kuin)

the number of solutions (ratkaisujen lukumäärä), one solution (yksi ratkaisu), “The point (a,b) is the solution to the pair/system of equations.” (“Piste (a,b) on yhtälöparin/yhtälöryhmän ratkaisu.”) / “The values $x = a$, $y = b$ is the solution to the pair/system of equations.” (“Yhtälöparin/yhtälöryhmän ratkaisu on $x = a$, $y = b$.”), infinitely many solutions (äärettömän monta ratkaisua), “This pair/system of equations is satisfied by all points on the line $y = kx + b$.” (“Kaikki suoran $y = kx + b$ pisteet toteuttavat yhtälöparin/yhtälöryhmän”), no solution (ei ratkaisua), “This pair/system of equation has no solution.” (“Tällä yhtälöparilla/yhtälöryhmällä ei ole ratkaisua.”)

the first (ensimmäinen), the second (toinen), the former (aiempi), the latter (jäljempinen)

to solve algebraically (ratkaista algebrallisesti), an algebraic solution (algebraallinen ratkaisu), the elimination method (yhteenlaskukeino), the substitution method (sijoituskeino), to find the value of (selvittää jonkin arvo), to substitute a for x (sijoittaa arvo a muuttujan x paikalle)

Sequences (Lukujonot)

a (number) sequence ((luku)jono), a pattern (kuvio / toistuva ominaisuus), to repeat (toistua), repetitive (toistuva / toistava), repetition (toisto), an element / a member (jäsen), the n th element / the n th member (n . jäsen)

an arithmetic sequence (aritmeettinen lukujono), the difference of consequent elements / members (peräkkäisten jäsenten erotus), a geometric sequence (geometrinen lukujono), the ratio / the quotient of consequent elements / members (peräkkäisten jäsenten suhde / osamäärä), constant (vakio)

Proportions and proportionalities (Verrannot ja verrannollisuus)

a ratio (suhde), the numerator (osoittaja) / the dividend (jaettava) / the antecedent (ensimmäinen jäsen), the denominator (nimittäjä) / the divisor (jakaja) / the consequent (jälkimmäinen jäsen), $a : b = c : d$ a is to b as c is to d ($a : b = c : d$ a :n suhde b :hen on yhtä suuri kuin c :n suhde d :hen), a proportion (verranto), the extremes of a proportion (a & d) (verrannon äärimmäiset jäsenet), the means of a proportion (b & c) (verrannon keskimmäiset jäsenet)

(to be) proportional to ((olla) verrannollinen johonkin), directly proportional to (suoraan verrannollinen johonkin), a ratio (suhde), a line (suora), inversely proportional to (kääntäen verrannollinen johonkin), a product (tulo), an inverse of (jonkin käänteisluku), a hyperbola (hyperbeli), (to be) constant (olla vakio), the constant of proportionality (verrannollisuuskerroin), a shortcut (oikotie), cross-multiplication (ristiin kertominen), to cross-multiply (kertoa ristiin), to clear fractions (päästä murtoluvuista eroon)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping examples of sequence patterns visible in the classroom.

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions, by colours or by symbols.

If an x unknown number is $\cdot 2$ doubled and then $+6$ six is added to that, you get $= 15$ fifteen. Work out the unknown number.

$$x \cdot 2 + 6 = 15 \quad || -6$$

$$x \cdot 2 = 15 - 6$$

$$x \cdot 2 = 9 \quad || :2$$

$$x = 9 : 2 = 4,5$$

Answer: The unknown number is 4,5.

Assigning reading tasks with automated formative pronunciation feedback on a digital learning platform (e.g. *Microsoft Teams*).

C4 Functions

This section does not directly correspond to any other content outcomes in previous grade brackets.

Content outcomes

C4 Functions

Correlations are depicted both graphically and algebraically.

The pupils

- familiarise themselves with direct and inverse proportionality.
- get acquainted with the concept of functions.
- draw straight lines and parabolae in a coordinate system.
- learn the concepts of the angular coefficient and the constant term.
- interpret graphs
- determine zeros of functions.

Language outcomes

The students practise:

- identifying the different possible meanings of notations based on the context
 - e.g. $2,3$ being an English list containing 2 and 3 or the Finnish decimal number two-point-three, and $(2,3)$ being the point $x = 2, y = 3$
- identifying notations with the same meaning
 - e.g. $f(2) = 3$ being the point $(2,3)$ and the intersection $x = 2, y = 3$
 - e.g. the notation $2 \leq x < 5$ being the same as x being 2 or in between 2 and 5.

The students practise:

- reading varying notations regarding functions
 - $f(x) = 2x - 3$ being read as *f of x is equal to 2 x minus three.*
 - $f(2) = 1$ being read as *f of two is equal to 1.*
 - $y = -5x + 2$ being read as *y is equal to negative five x plus two.*
 - $g(x) = -7x^2$ being read as *g of x is equal to negative seven x squared.*
- describing properties of a function based on its expression
 - e.g. the constant term for $f(x) = 2x - 3$ is *negative three*
 - e.g. the y -intercept for $y = -5x + 2$ is at *x is zero, y is two*
 - e.g. the angular coefficient of $y = -5x + 2$ is *negative five*
- categorising a function based on its graph, its expression, or a list of its values
 - e.g. being increasing or decreasing; intersecting axes or other graphs; being linear, quadratic or otherwise; representing direct proportionality, inverse proportionality or neither

The students practise:

- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- giving constructive feedback on a pupil's, a teacher's or an AI's thought process, and comparing it to their own approach
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- evaluating, assessing, and reasoning whether the obtained value is appropriate for the assignment, task or problem.

Attachments and links

Unit: *Intro to slope* by Khan Academy,
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[Speaking frame AB exercise:](#)
[Reading function notations](#)

Speaking frame on reading function expressions
([printable](#))

Key words

a coordinate system (koordinaatisto), an axis (akseli), the x -axis (x -akseli), the y -axis (y -akseli), the xy -coordinate system (xy -koordinaatisto), a coordinate plane (tasokoordinaatisto), a coordinate (koordinaatti), the origin (origo), the 1st / I quadrant (1. / I neljännes), the 2nd / II quadrant (2. / II neljännes), the 3rd / III quadrant (3. / III neljännes), the 4th / IV quadrant (4. / IV neljännes), an ordered pair of numbers (järjestetty lukupari), an x -coordinate (x -koordinaatti), a y -coordinate (y -koordinaatti), a point (piste)

a function (funktio), a notation (merkintä), an expression (lauseke), the degree (aste), a term (termi), a constant (vakio), constant term (vakiotermi), a coefficient (kerroin), a slope (kulmakerroin), a variable (muuttuja), a value (arvo), to substitute (sijoittaa), to plot (piirtää kuvaaja), a graph (kuvaaja), a line (suora), a parabola (paraabeli), a hyperbola (hyperbeli)

an intersection (leikkauspiste), to intersect (leikata), a zero (nollakohta), a root (juuri), a maximum (suurin arvo), a minimum (pienin arvo), an x -intercept (x -akselin leikkauspiste), a y -intercept (y -akselin leikkauspiste)

to increase (kasvaa), to be increasing (olla kasvava), to decrease (laskea), to be decreasing (olla laskeva), to be positive (olla positiivinen), to be negative (olla negatiivinen), an ascending line (nouseva suora), descending line (laskeva suora)

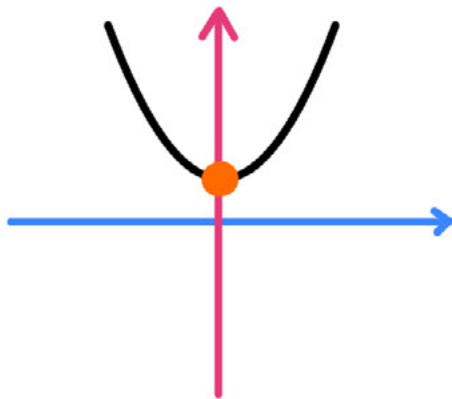
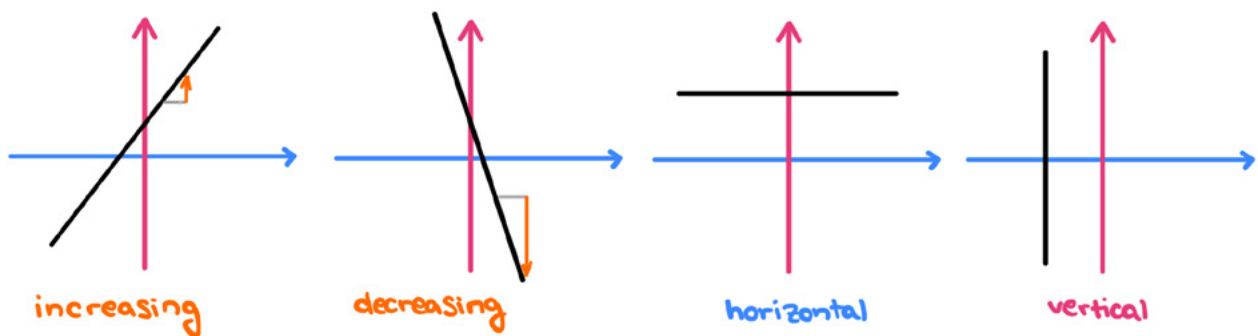
directly proportional to (suoraan verrannollinen johonkin), inversely proportional to (kääntäen verrannollinen johonkin)

Scaffolding

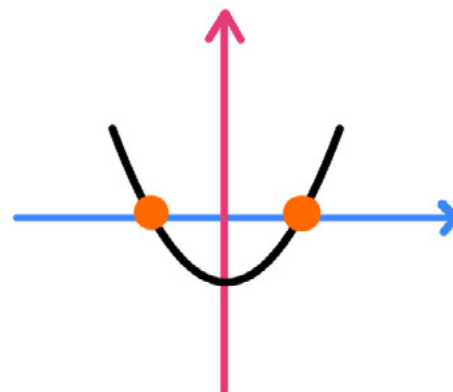
Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Keeping examples of function graph types, together with their main properties visible and named in the classroom.



The **vertex** of a parabola $y = ax^2 + c$ is at its **y-intercept**: $(0, c)$.

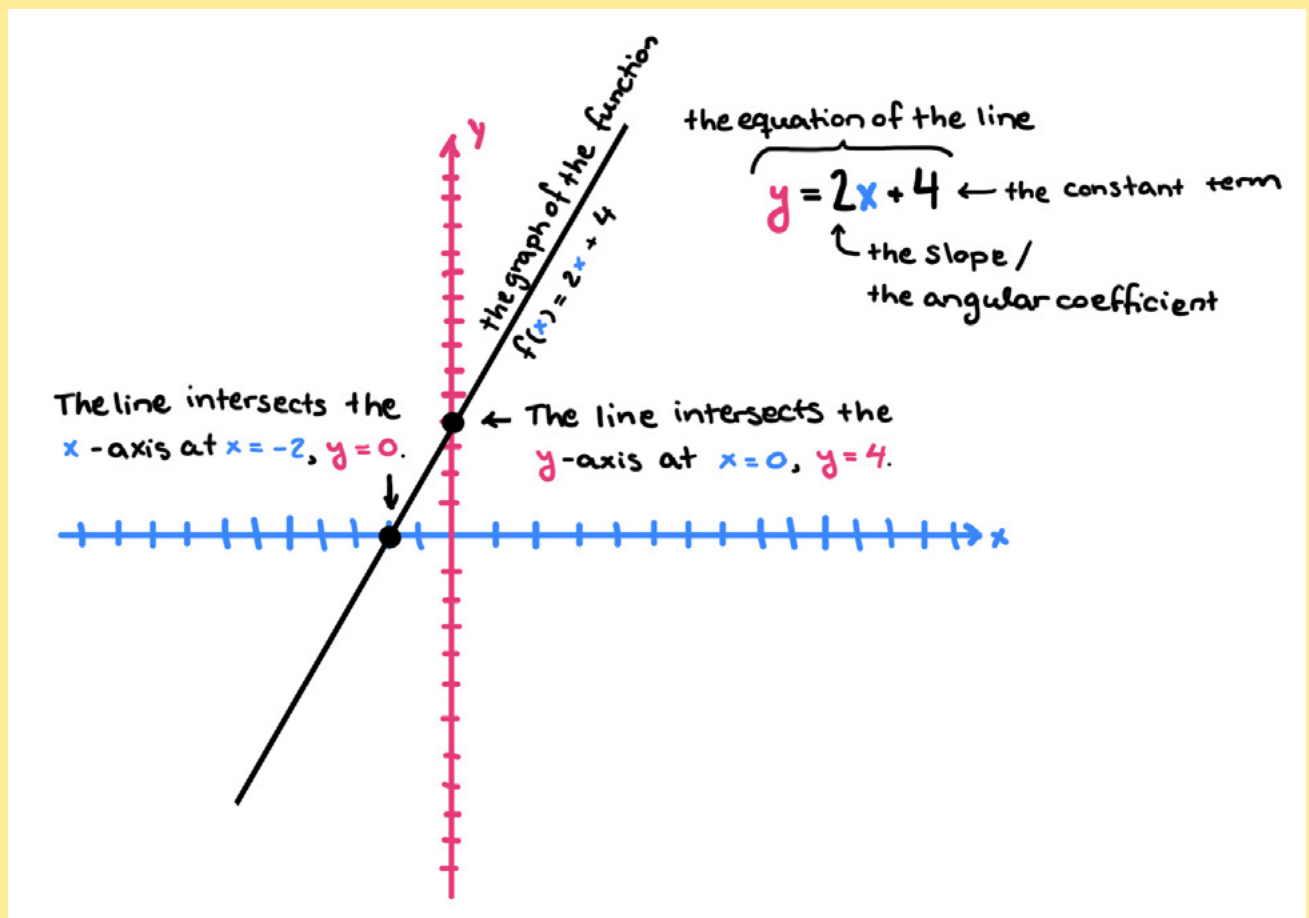


The **zeros** of the 2nd degree polynomial $f(x) = ax^2 + c$ coincide with the **x-intercepts** of the graph $y = ax^2 + c$.

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions (rf. to output scaffolding methods in C3 and C5 for grades 7–9).

Offering students with sentence diagrams to describe the main features of graphs.



C5 Geometry

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C3 for grades 1–2, and C4 for grades 3–6.

Content outcomes

C5 Geometry

The students

- expand their understanding of the concepts of a point, a line segment, a straight line, and an angle
- familiarise themselves with the concepts of the line and ray.
- examine properties connected to lines, angles, and polygons.
- reinforce their understanding of the concepts of similarity and congruence.
- practise geometric construction.
- learn to use the Pythagorean theorem, the converse of the Pythagorean theorem, and trigonometric functions.
- learn about the inscribed angle and the central angle and familiarise themselves with the Thales' theorem.
- calculate the circumferences and areas of polygons.
- practice calculating the area, length of the circumference, and the area and the area of a sector of a circle.
- examine three-dimensional figures and learn to calculate the areas and volumes of the sphere, the cylinder, and the cone.

Language outcomes

Listening skills

The students practise:

- identifying terminology and vocabulary related to geometry and measuring from spoken Finnish and English, and acting according to detailed oral instructions of various levels of complexity.
 - e.g. “Draw a circle whose radius is 3 centimetres.”
 - e.g. “You will need your protractor in the next assignment, in order to draw an isosceles obtuse triangle whose area is ten centimetres squared.”
 - e.g. “Sketch an acute angle and an obtuse angle.”
 - e.g. “Open your notebook. Write down the Pythagoras' theorem formula.”

Reading skills

The students practise:

- identifying mathematical terminology and vocabulary in written Finnish and English, and acting according to detailed written instruction.
- identifying parts of shapes and figures by their written names in Finnish and in English.
 - e.g. “The base of this figure is a circle, and there is a right angle there, so this must be a right circular cone.”
 - e.g. V representing volume, p representing perimeter, h representing height, A representing area (rf. Attachments and links).

Speaking, writing, and illustrating skills

The students practise:

- classifying geometric shapes and figures and explaining their key properties by using their own words
 - e.g. “A cuboid has six faces, twelve edges, and eight vertices.”
 - e.g. “This trapezoid can be broken down into a rectangle and two right triangles.”
- verbalising their thought process when carrying out measurements.
 - e.g. “I placed the middle part of the protractor here at the vertex and then read the degrees over here.”

- rephrasing assignments in their own words.
 - e.g. “To build this right circular cylinder out of paper, I need to cut out two congruent, identical circles and a rectangle.”
- using words with irregular plural forms
 - e.g. a vertex, many vertices; a radius, many radii
- reading terminology and vocabulary with loaned prefixes and suffixes
 - e.g. tri- representing 3, quadri- representing 4, and penta- representing 5, respectively.
 - e.g. -lateral representing sides, and -sect representing cutting.
- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
 - e.g. “Instead of searching for a direct solution, we (should / could) try calculating the area of this trapezoid by calculating the area of these two triangles and of this rectangle, and then adding them up.”
 - e.g. “There are many ways to find a solution in mathematics.”
 - e.g. “What is the sum of the angles of a triangle?”
 - e.g. “Angles in a triangle add up to 180°. Can you use this to calculate the inscribed angle?”
 - e.g. “The formula for the surface area of a right circular cone can be derived effortlessly: you only have to substitute the perimeter of the base circle to the arc length formula to obtain the expression for the central angle.”
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- giving constructive feedback on a pupil’s, a teacher’s or an AI’s thought process, and comparing it to their own approach.
- assessing and reasoning if the value they get fits the assignment, task or problem.
 - e.g. “As the triangle is inside the square, its area should not be greater than the area of the square.”

The students practise:

- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- giving constructive feedback on a pupil’s, a teacher’s or an AI’s thought process, and comparing it to their own approach
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- evaluating, assessing, and reasoning whether the obtained value is appropriate for the assignment, task or problem.

Attachments and links

[Classifying quadrilaterals](#)

[Classifying triangles](#)

[Calculations on squares](#)

[Calculations on rectangles](#)

[Calculations on parallelograms](#)

[Calculations on triangles](#)

Inscribed and circumscribed angles

(<https://www.geogebra.org/m/hcckyx5>)

[GeoGebra assignments on planar geometry](#)

Area of a circle, formula explained

(<https://www.youtube.com/watch?v=YokKp3pwVFc>)

An example of a glossary in table form: relevant terms in both Finnish and in English, together with their respective explanations in English.

suomeksi	in English	explanation in mathematics	explanation in layman's terms
piste	point	An object with no width, height, or depth.	Drawn as a dot with a pen.
jana	line segment	An object with length only, straight, with both a starting point and an end point.	Drawn with a ruler, a dot or a dash on both ends.
puolisuora	ray	An object with length only, straight, with a starting point but no end point.	Drawn with a ruler, a dot or a dash on one end.
suora	line	An object with length only, with no starting or end point.	Drawn with a ruler.
viiva	curve	An object with length only, with no starting or end point.	Drawn freehand.
murtoviiva	polygonal chain	An object consisting of line segments, whose end points are starting points for consequent line segments.	Drawn with a ruler, by rotating it in between drawing the line segments in succession.
taso	plane	An object with width and depth, flat.	The page in your notebook.

An example of a glossary in table form: relevant terms in both Finnish and in English, together with their respective explanations in English.

suomeksi	in English	explanation
kehä	circumference	The length of the circle.
keskipiste	centre	The point that is equidistant from all points on the circle.
säde	radius (plural: radii)	A line segment that connects the centre and the circle.
jänne	chord	A line segment that connects two points on the circle.
halkaisija	diameter	A chord that passes through the centre.

suomeksi	in English	explanation
keskuskulma	central angle	The angle between two radii.
kehäkulma	inscribed angle	The angle between two chords with a common end point.
kaari	arc	A section of a circle.
tangentti	tangent	A line that passes through the circle once and only once.
tangenttikulma	angle between tangents	An angle between two tangents.

A table of the Latin alphabet with the equivalent of each letter in the Greek alphabet.

Latin alphabet	Greek alphabet	in Finnish	in English
a	α	alfa	alpha
b	β	beta	beta
c	γ	gamma	gamma
d	δ	delta	delta
e	ε	epsilon	epsilon
f	φ	fii	phi
h	η	eta	eta
th	θ	theta	theta
i	ι	iota	iota
k	κ	kappa	kappa
l	λ	lambda	lambda
m	μ	myy	mu
n	ν	nyy	nu
o	ω	omega	omega
p	π	pii	pi
r	ρ	roo	rho
s	σ	sigma	sigma
t	τ	tau	tau
u,y	υ	upsilon	upsilon
x	ξ	ksii	xi
z	ζ	zeta	zeta

A table summarising the quantities in geometry, together with their one-letter abbreviations and with their names in English and in Finnish.

abbreviation	original	Finnish translation
V	volume	tilavuus
A	area	pinta-ala
h	height	korkeus
p	perimeter	piiri
d	diameter	halkaisija
r	radius	säde
s	slant height	sivujana
a, b, c	Used to denote side length.	
x, y, z	Used to mark denote side length.	

Key words

a point (piste), a line (suora), a plane (taso), a solid (kappale)

a dimension (ulottuvuus), length (pituus), width (leveys), depth (syvyys), height (korkeus)

to sketch (hahmotella), to draw (piirtää), to illustrate (havainnollistaa)

Lines and curves (Suorat ja viivat)

a curve (viiva), a (straight) line (suora), a ray (puolisuora), a starting point (alkupiste), an end point (päätepiste), a line segment (jana), a polygonal chain (murtoviiva), a secant (sekantti), a tangent (tangenti), an intersection (leikkaus), to intersect (leikata), transversal/intersecting lines (leikkaavat suorat)

to be orthogonal to/perpendicular to (olla kohtisuorassa jonkin suhteen), a normal (normaali), to bisect (puolittaa), a bisector (puolittaja/keskijana), a perpendicular bisector (keskinormaali), to be parallel to (yhdensuuntainen), to be identical to (yhtenevä)

Angles (Kulmat)

a side (kylki), a vertex (*many vertices*) (kärki)

a zero angle (nollakulma), an acute angle (terävä kulma), a right angle (suora kulma), an obtuse angle (tylppä kulma), a convex angle (kovera kulma), a straight angle (oikokulma), a reflex angle (kupera kulma), a full angle/rotation (täysikulma)

a degree (aste), a radian (radiaani), a gradian/grad/grade (graadi)

opposite/vertical angles (ristikulmat), adjacent angles (vieruskulmat), alternate/corresponding angles (samankohtaiset kulmat), to bisect (puolittaa),

a bisector (kulmanpuolittaja)

Polygons and planar geometry (Monikulmiot ja tasogeometria)

two-dimensional (kaksiulotteinen), a vertex (*many vertices*) (kärki), a side (sivu), a diagonal (lävistäjä), closed non-transversal polygonal chain (suljettu itseään leikkaamaton murtoviiva)

a base (kanta), a height (korkeus), a perimeter (piiri), a circumference (kehän pituus), an area (pinta-ala)

a triangle (kolmio), a scalene triangle (epäsäännöllinen kolmio), an isosceles triangle (tasakylkinen kolmio), an equilateral triangle (tasasivuinen kolmio), a leg (kylki), a base angle (kantakulma), an apex angle (huippukulma), an acute triangle (teräväkulmainen kolmio), an obtuse triangle (tylppäkulmainen kolmio), a median (keskijana), a centroid/a triangle centre/center (kolmion keskipiste), an inscribed circle (kuvion sisään piirretty ympyrä), a circumscribed circle (kuvion ulkopuolelle piirretty ympyrä)

a right triangle (suorakulmainen kolmio), a hypotenuse (hypotenuusa), a leg/cathetus (many catheti) (kateetti), the Pythagorean theorem/Pythagoras' theorem (Pythagoraan lause)

a quadrilateral/quadrangle (nelikulmio), a trapezium/trapezoid (puolisuunnikas), a parallelogram (suunnikas), a rhombus (neljäkäs, vinoneliö), a rectangle (suorakulmio), a square (neliö),

a polygon (monikulmio), a regular polygon (säännöllinen monikulmio), a pentagon (viisikulmio), a hexagon (kuusikulmio), a heptagon (seitsenkulmio), an octagon (kahdeksankulmio), a nonagon (yhdeksänkulmio), a decagon (kymmenkulmio), an n-gon (n-kulmio)

a circle (ympyrä), a centre (keskipiste), a radius (many radii) (säde), a diameter (not diametre) (halkaisija), a chord (jänne), a central angle (keskuskulma), an arc (kaari), an angle subtended by an arc/ corresponding angle (kaarta vastaava keskuskulma), the length of an arc (kaaren pituus), an inscribed angle (kehäkulma), a semicircle (puoliympyrä), a sector (sektori), a segment (segmentti), a secant (sekantti), a tangent (tangenti), a tangential angle (tangenttikulma), an annulus (many annuli) (rengas)

a figure (kuvio), congruent to (yhtenevä jonkin kanssa), similar to (yhdenmuotoinen jonkin kanssa), a corresponding point/side/angle (vastinpiste/sivu/kulma), a scale (mittakaava), to scale to (suurentaa/ pienentää johonkin), a ground plan (asemapiirros), a map (kartta), a blueprint (suunnitelmapiirros), to enlarge (suurentaa), enlarged (suurennettu), to reduce (pienentää), reduced (pienennetty), a ratio (suhde)

symmetry (symmetria), to be symmetrical with respect to a line/a point (suoran/pisteen suhteen symmetrinen), an axis of symmetry (symmetria-akseli), a centre/ center of symmetry (symmetriakeskus),

reflectional symmetry (peilaus), line symmetry (symmetria suoran suhteen), pointwise symmetry (symmetria pisteen suhteen)

a rotation around a point (kierto pisteen ympäri), to rotate around a point (kiertää pisteen ympäri), a translation (siirto), to translate (siirtää), to reflect across a line (peilata suoran suhteen), a reflection across a line (peilaus suoran suhteen), to reflect across a point (peilata pisteen suhteen), a reflection across a point (peilaus pisteen suhteen)

Solids and spatial geometry (Avaruuskappaleet ja avaruusgeometria)

three-dimensional (kolmiulotteinen), volume (tilavuus)

a perspective (perspektiivi), Cavalier's perspective (kavaljeeriperspektiivi), vanishing point perspective (pakopisteperspektiivi), isometric perspective (isometrinen perspektiivi)

a rotational solid (pyörähdyskappale), to rotate around an axis (pyörähtää akselin ympäri)

a base (pohja), length (pituus), width (leveys), depth (syvyys), height (korkeus)

a vertex (many vertices), an edge (särmä), a face/ facet (tahko), a surface/lateral area/lateral surface (vaipan pinta-ala), a net/total area (kokonaispinta-ala), a (space) diagonal ((avaruus)lävistäjä), an apex (huippu), right (suora), oblique (vino)

a cylinder (lieriö), a polyhedron (monitahokas), a prism (särmä), a cube (kuutio), a cone (kartio), height (h) (korkeus), slant height (s) (sivujana), a pyramid (pyramidi), a tetrahedron (tetraedri), a sphere (pallo)

Plato (Platon), a Platonic solid (Platonin kappale), tetrahedron (tetraedri), hexahedron (cube) (kuutio), octahedron (oktaedri), dodecahedron (dodekaedri), icosahedron (ikosaedri)

Trigonometric functions (Trigonometriset funktiot)

sine (sini), cosine (kosini), tangent (tangenti),
opposite (vastainen), adjacent (viereinen), hypotenuse
(hypotenuusa), leg/cathetus (*many catheti*) (kateetti),
incline (jyrkkyys, kaltevuuskulma), steep (jyrkkä),
elevation (korkeus), horizon (horisontti, taivaanranta),
inverse (käänteisfunktio)

Units of measurements (Mittayksiköt)

unit conversions (yksikönmuunnokset),
to convert from ... to ... (muuntaa jostakin johonkin)

-metre (UK) / -meter (US) (-metri)

millimetre (millimetri), centimetre (senttimetri),
decimetre (desimetri), metre (metri), decametre
(dekametri), hectometre (hektometri), kilometre
(kilometri)

square metre/metre squared (neliömetri)

square millimetre (neliömillimetri), square centimetre
(neliösenttimetri), square decimetre (neliödesimetri),

square metre (neliömetri), are (aari), hectare
(hehtaari), square kilometre (neliökilometri)

cubic metre/metre cubed (kuutiometri)

cubic millimetre (kuutiomillimetri), cubic
centimetre (kuutiosenttimetri), cubic decimetre
(kuutiodesimetri), cubic metre (kuutiometri), cubic
decametre (kuutiodekametri), cubic hectometre
(kuutiohektometri), cubic kilometre (kuutiokilometri)

-litre (UK) / -liter (US) (-litra)

millilitre (millilitra), centilitre (senttilitra), decilitre
(desilitra), litre (litra)

Equipment

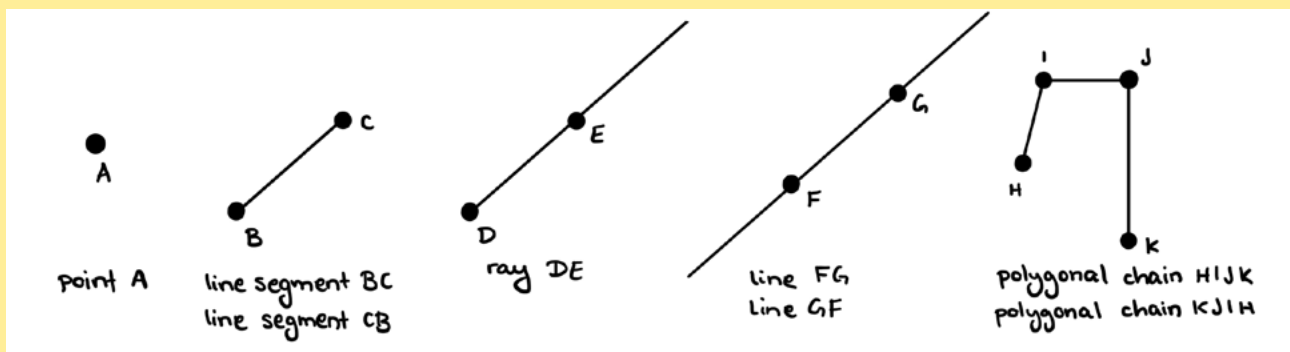
a ruler (viivain/viivoitin), a protractor (kolmioviivain/
geokolmio/astemitta), a compass (harppi), a (scientific)
calculator ((tieteellinen) laskin), the All Clear button AC
(tyhjennä kaikki -painike AC)

geometrical construction (geometrinen piirtäminen),
to construct geometrically (piirtää geometrisesti), with
a straightedge and a compass (harpilla ja viivaimella)

Scaffolding

Input scaffolding

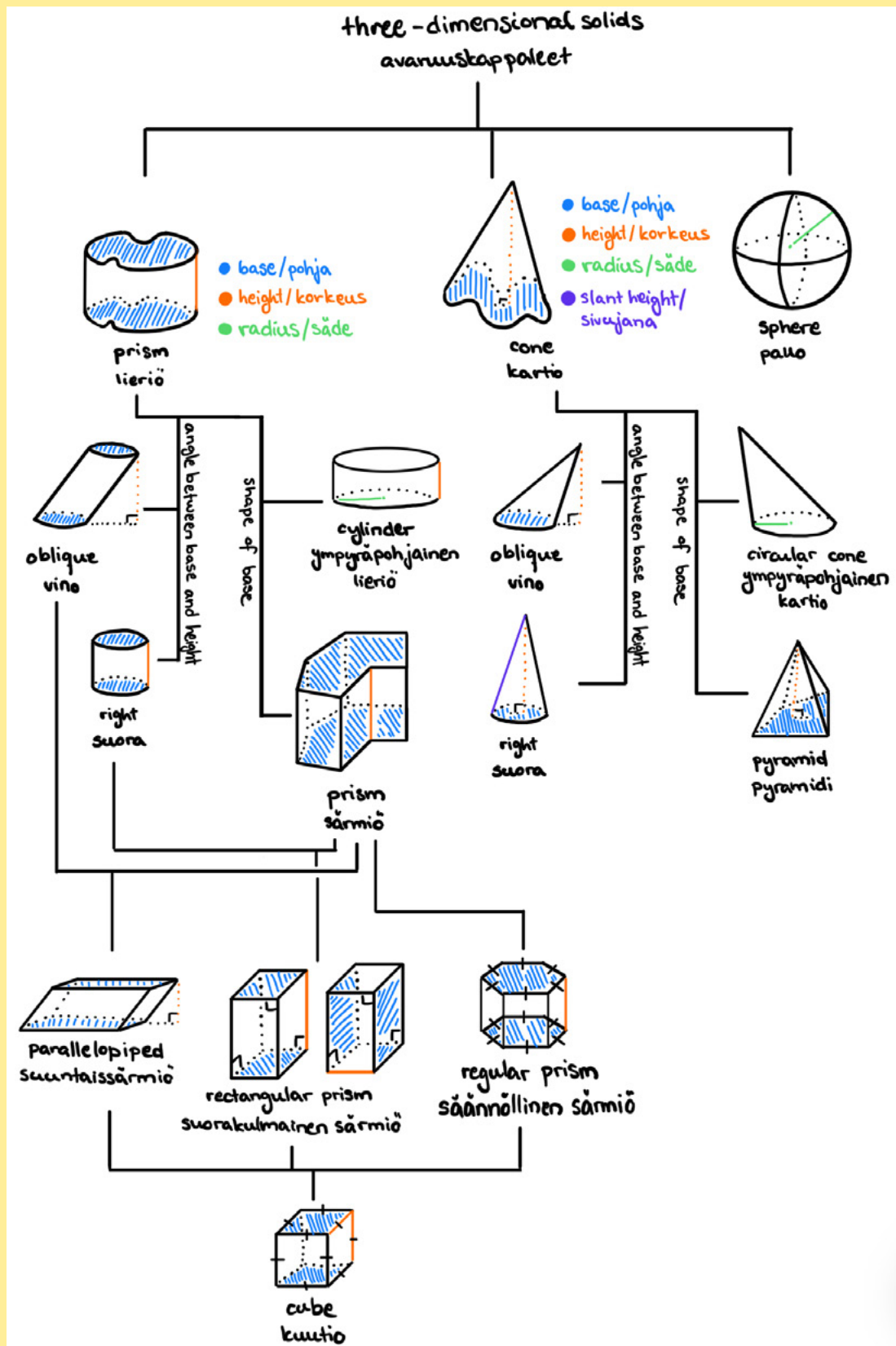
Keeping shapes, their names, and key characteristics visible in the classroom throughout.



Creating a glossary chapter-by-chapter (rf. Attachments and links of this section for an example).

Using dynamic geometry software, e.g. GeoGebra, to model theorems, e.g. inscribed and circumscribed angles (rf. Attachments and links of this section for both a simulation example and ready-made assignments on GeoGebra).

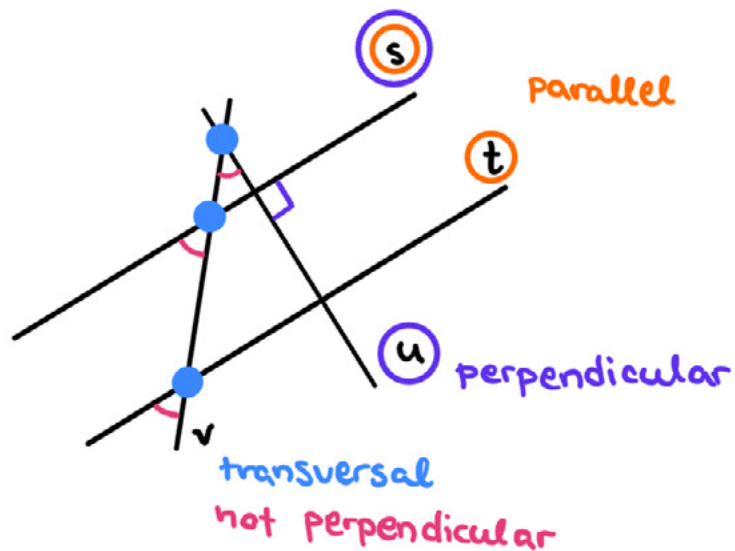
Comparing new geometric shapes to ones learned previously.



Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions.

Draw lines s , t , u , and v so that s is parallel to t , s is perpendicular to u , $s \parallel t$, and that $s \perp u$, and that line v is transversal to all others but no right angles to any of the three.



C6 Data processing, statistics, and probability

For additional material, scaffolding methods, and language outcomes related to the same subject-content, please refer to the following entries: C4 for grades 1–2, and C5 for grades 3–6.

Content outcomes

C6 Data processing, statistics, and probability

The students

- deepen their skills in collecting, structuring, and analysing data.
- deepen their understanding of the average and mode.
- practise defining frequency, relative frequency, and median.
- familiarise themselves with the concept of dispersion.
- interpret and produce different diagrams.
- calculate probability.

Language outcomes

The students practise

- reading relevant details – averages and medians; frequencies; maximal and minimal values; increasing and decreasing trends; and dispersion – from various data representations – tables, maps, charts, graphs and the like.
- assessing which representations best suit the given data source and the assignment.
- forming clear and unambiguous data collection questions to ensure consistent responses.
- using terminology and vocabulary from other subjects in the context of statistics.
- assessing the reliability of data sources.
- classifying events as almost impossible, improbable, probable, possible or almost certain.
- calculating the number of events – a number being even or odd, a card being of a given suit, and similar events – in a given context – for a die, a deck of cards, and similar scenarios.
- reading notations with an unspecified number of elements, e.g. in a sigma notation or a formula
- rephrasing the relative sizes of events in fraction and decimal notation.

The students practise:

- comparing different approaches to an assignment, task or problem, and assessing and reasoning which one best suits the scenario, using correct terminology and vocabulary.
- giving constructive feedback on a pupil's, a teacher's or an AI's thought process, and comparing it to their own approach
- presenting their answer so it can be understood by pupils and teachers, and backing up their reasoning with concise arguments, clearly written intermediate steps, and relevant drawn figures.
- evaluating, assessing, and reasoning whether the obtained value is appropriate for the assignment, task or problem.

Attachments and links

[A deck of cards](#)

[Klassinen todennäköisyys](#)

[Different graph types with context & Add a chart or graph](#), on Google Docs Help Center

[Different graph types with context & Add a chart or graph](#), on Microsoft Support

Key words

Statistics (Tilastot)

the average (keskiarvo), the mean (keskiluku), the arithmetic mean (aritmeettinen keskiarvo), the geometric mean (geometrinen keskiarvo), the frequency (frekvenssi), the relative frequency (suhteellinen frekvenssi), the median (mediaani), a decile (desiili), a percentile (persentiili), the mode (moodi), a weighted average (painotettu keskiarvo)

to be increasing (olla kasvava), to be decreasing (olla vähenevä), a maximal value (suurin arvo), the maximum (suurin arvo), a minimal value (pienin arvo), the minimum (pienin arvo), to disperse (hajota), dispersion (hajoaminen), to deviate (poiketa), deviation (poikkeama), to vary (vaihdella), variance (variassi / vaihtelevuus), a range (vaihteluväli)

Charts (Kaaviot)

a graph (kaavio / kuvaaja), a chart (kaavio / kuvaaja), a bar graph (pylväsdiagrammi), a pie graph (piirakkakaavio), a column chart (sarakekaavio), a bar chart (pylväsdiagrammi), a scatterplot (hajontakuvaaja), a table (taulukko), a column (sarake), a row (rivi), a cell (solu), a spreadsheet file (taulukkolaskentatiedosto)

a function (funktio), a range (määritetyt solut), to compute (laskea laskennallisesti), to calculate (laskea laskutoimitus)

Statistical studies (Tilastolliset tutkimukset)

a population (populaatio), a sample (otos), to represent (edustaa), to select (valita), to observe (havaita), a questionnaire (kysely), a form (lomake), to gather data (kerätä tietoa), to analyze data (analysoida tietoa)

Probability measures (Todennäköisyyden mittaaminen)

an event (tapahtuma), a feasible event (mahdollinen tapahtuma), probable (todennäköinen), almost surely (lähes varmasti), almost never (ei lähes milloinkaan), a geometric probability (geometrinen todennäköisyys), a statistical probability (tilastollinen todennäköisyys)

Contexts for probability (Todennäköisyyden havainnollistaminen)

a die (*many dice*) (noppa), to roll a die (heittää noppaa), to roll dice (heittää noppia), a face of a die (nopan tahko), a pip (silmäluku), a D6 die (D6-noppa), a DN die (DN-noppa) ($N = 4, 5, \dots$)

a coin (kolikko), heads (kruuna, the symbolic side), tails (klaava, the numerical side), to flip a coin (heittää kolikkoa), to turn up/get heads (saada kruuna), to turn up/get tails (saada klaava)

a deck of cards (korttipakka), a suit (maa), spades ♠ (pata), hearts ♥ (hertta), diamonds ♦ (ruutu), clubs ♣ (risti), a face card / a court card (kuvakortti), an ace (ässä), a jack (jätkä), a queen (kuningatar), a king (kuningas)

Scaffolding

Input scaffolding

Regularly revisiting concepts, terminology, vocabulary, and notations through repetition.

Tying in new statistical concepts to subject matter in other subjects, e.g. geography.

Output scaffolding

Breaking down assignments into easy-to-follow, step-by-step instructions (rf. to output scaffolding methods in C3 and C5 for grades 7–9).

Offering students with sentence diagrams they can fill in to interpret graphs and read data in table format.